

A STRATEGY FOR PRODUCING HYBRID REGIONAL INPUT-OUTPUT TABLES*

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1. INTRODUCTION

Hybrid input-output models combine nonsurvey techniques for estimating regional direct requirements tables with superior data, which are obtained from experts, surveys, and other reliable sources (primary or secondary). Such data can be added at any stage of model construction. To date, hybrid model production has been defined only in very general terms. Elsewhere (Lahr, 1993), I have pointed out directions that could prove fruitful in developing a more detailed approach.

I start the paper by searching for an approach that recognizes critical portions of input-output models that are ripe for survey work or the collection of other superior data. This search begins with a review of existing measures that identify *sectors* that are most critical to the accuracy of the Leontief inverse. I next compare these measures for their ability to predict sectors critical to the survey-based 1972 Washington State Input-Output Table.

In Section 3, I combine the findings of Section 2 with that of other literature to develop a quasi-complete superior-data search strategy for producing hybrid regional input-output tables. In Section 4, I test the strategy by employing the 1972 Washington State input-output table. Tests are of models both closed and open with respect to households. Following this, I conclude with implications of the findings.

2. IDENTIFYING CRITICAL SECTORS FOR SUPERIOR-DATA COLLECTION

The best-known measure (Jensen and West, 1980; Hewings and Romanos, 1981; West, 1981) used to build hybrid regional input-output models identifies *cells* that are most critical to the accuracy of output multipliers. [For a recent contribution and nearly complete review of this line of literature, please see Giarratani and Garhart (1991), Jackson (1991), or Casler and Hadlock (1997).] There is some doubt, however, about the practicality of such a cell-by-cell approach when used as a means of identifying the portions of input-output tables that could make its Leontief inverse more accurate. The rationale behind these objections is that obtaining superior data for a single cell in a given sector may not be worth the effort. This is especially the case when data for a collection of cells in the same or another sector may be much more valuable and require nearly the same effort. In support of this, for example, Isard and Langford (1971) inform us that data on regional imports for individual cells are much more difficult to obtain than they are by sector (either rowwise or columnwise). Furthermore a set of literature (Miernyk, 1970; Bourque, 1971; Conway, 1975; Afrasiabi and Casler, 1991) has established that proper accounting of trade patterns is critical to the accuracy of regional input-output models. It, therefore, follows that identifying sectors rather than individual cells as targets of survey work is likely to prove more cost effective.

By “important sector,” model constructors, generally mean sectors that are most critical to model accuracy in everyday use. The measures that traditionally have been used to identify

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such sectors are known as “key sector” measures. Key sectors are sectors that have the most total linkages in an economy (Cella, 1984). Currently, the most accepted key-sector measures are those that combine traditional Leontief and Ghoshian linkages and those that employ hypothetical extraction techniques. In comparing these measures, Lahr (1992) investigates the theoretical inadequacies and the empirical impracticalities of those that combine Leontief and Ghoshian linkages (Jones, 1976; Beyers, 1976; Hübler, 1979; Loviscek, 1982). He suggests that, of the two, hypothetical extraction (Paelinck et al., 1965; Miller, 1966; Meller and Marfàn, 1981; Cella, 1984; Szyrmer, 1992) should be favored. Hypothetical extraction measures are calculated by finding the effects of eliminating a sector from the economy (c.f., Lahr and Miller, 1997). More recently, de Mesnard (1997) and Dietzenbacher (1997) have provided evidence that should rekindle interest in a hybrid Leontief-Ghoshian, hypothetical-extraction approach. The problem of how to combine such Leontief and Ghoshian linkages remains, however.

In the specific case for this paper by “important sector” I really mean a sector for which superior data will significantly improve nonsurvey model accuracy. Sectors that have technology that is very different from that represented in the nonsurvey model and that also have large total linkages will be identified correctly by hypothetical extraction approaches. But what about sectors for which the technology is somewhat well represented by the nonsurvey model but for which total linkages are large? The same goes for sectors whose technology is very different from that in the nonsurvey model but which have total linkages that are medium to small in magnitude. Unfortunately, hypothetical extraction approaches are not designed to ascertain such differential importance of changing the technology of sectors. Instead, they measure error only due to the total elimination of a sector, not increases or decreases in its interconnectivity with the rest of the economy through differences in technology or trade patterns.¹

The reason we are interested in differences in technology and trade patterns is that literature on the accuracy of regional input-output has shown that differences in trade patterns and technology account for most of the differences between the direct requirements matrices of a nation and each of its component regions. (For a review see, Giarratani and Garhart, 1991.) The most popular nonsurvey regionalization approaches strictly adjust for imports by producing sector, working on rows of national direct-requirements matrices, by assuming that technology in the nation is spatially invariant. This approach has proven somewhat accurate. Subsequently, when nonsurvey regional direct-requirements matrices are multiplied by the diagonal matrix of outputs, reasonable estimates of intermediate outputs by sector result.

Through similar matrix operations, however, very reasonable estimates for intermediate inputs by sector are not necessarily to be expected. This is because by regionalizing strictly across rows, imported proportions of intermediate inputs are not well estimated. That is, the use of imports by industry is likely to be badly estimated by most nonsurvey models. Consequently, the problem of identifying “important sectors” for the most common situation is one of identifying sectors that are likely to induce the largest *changes* in total linkages when survey data replace nonsurvey data for the sector’s direct requirements. Essentially then, we are interested in determining the effects on the Leontief inverse of changes in a single sector’s direct requirements. This problem is answered through a line of literature that started as early as Sherman and Morrison (1949, 1950) and Woodbury (1950).

The Effects of Intermediate-Input Variation on the Leontief Inverse

¹ Furthermore, linkages are highly nonlinear (West 1981, 1982). Hence, by eliminating an entire sector we would undoubtedly be miscalculating the measure that we need.

Sell (1980) tells us, and as mentioned earlier, that a strong direct relationship exists between the magnitude of the proportion of intermediate inputs and the potential of the sector to generate error in its output multiplier. This suggests that a measure similar to that of West (1981) but extended to the vector of intermediate inputs might be a reasonable one to use. Sell, using a national model, was working on the assumption that (near) perfect information on imports and exports by producing sector is known. This assumption regarding trade patterns is overly strong for prototypical hybrid regional models, generally nonsurvey models are formed by the rows-only regionalization of national technology. And even the data need to produce the trade data for rows are typically estimated.

West's (1981) cell-sensitivity formula includes a proportional perturbation factor. That is, unlike the hypothetical extraction approach, it allows the analyst to adjust the proportion by which a cell is altered to effect a change in all output multipliers. By continuing the assumption of spatially invariant national technology, these proportions can be viewed as the error in coefficient estimates that are due to the mis-estimation of the use of imports by sector. Therefore, if West's (1981) formula could be generalized in some way to apply to the column of each sector (intermediate inputs), the solution to our problem would seem to have been found. Fortunately, such a generalized formula was provided as early as Evans (1954) and Dwyer and Waugh (1953) and as recently as Maaß (1980) and West (1982). The derivation of Evans's (1954) version of the formula is provided in the Appendix A. The final formulation of this variant is the most efficient to calculate and can be expressed as

$$(1) \quad \mathbf{E} = [(\mathbf{I} - \mathbf{BP})^{-1} - \mathbf{I}] \mathbf{B}$$

where \mathbf{B} is the Leontief inverse, \mathbf{I} is an identity matrix, and \mathbf{P} is the matrix of multiplicative perturbations of the direct-requirements matrix.

To find out the effect that a percentage change in a sector's information has on the rest of the economy, all cells of \mathbf{P} should be set to zero except for the column and row for which we want to test sensitivity to imports and calculate \mathbf{E} . By letting $\mathbf{P} = [p_{ij} a_{ij}]$ (where the a_{ij} 's are the elements of the direct-requirements matrix), the values of the nonzero p_{ij} 's can be set to proportions that represent likely deviations of each of the sector's direct-requirement coefficient from its true value.

One disadvantage of \mathbf{E} is its matrix form. Comparing the \mathbf{E} matrices for each sector remains rather intractable (see an explanation in Appendix C). If each sector's error matrix could be expressed as a scalar, comparison of the importance of sectors would be much simpler. A means of creating such a scalar would be to pre-multiply \mathbf{E}_i , the error matrix for sector i , by a transposed vector of ones, $\mathbf{1}$, and to post-multiply by a vector of regional outputs (value added), \mathbf{x} , or some other set of economic weights like final demand, earnings, or employment

$$(2) \quad E_i = \mathbf{1}' \mathbf{E}_i \mathbf{x}$$

or in matrix form

$$(3) \quad \mathbf{E} = \mathbf{E} \mathbf{x}$$

3. A SIMPLE EMPIRICAL TEST OF THE MEASURE'S VALIDITY

Although the measure in the last section has appeal, there is no evidence that it can be valuable in targeting sectors for superior-data collection when estimated from a nonsurvey model. Furthermore, although it may seem conceptually sound to assume that column-only perturbations will provide reasonable sectoral rankings, I have not offered evidence to show that the addition of row perturbations of the same sector will not provide significantly more accurate rankings. Hence, I decided to test how well sectors can be targeted with the measure by comparing the results of the measure using data strictly from a nonsurvey model to those of the total linkage difference² between the nonsurvey and survey-based models. Naturally, sectors with the largest total linkage differences between survey-based and nonsurvey models are those that should be targeted for superior-data collection efforts. Consequently, if we find that by using E in Equation (3) we can obtain a ranking of sectors that is very similar to that arrived through an intersectoral comparison of the total linkage difference between the survey-based and nonsurvey models, we should deem E useful in this instance.

I used the survey-based 1972 Washington State Input-Output Table closed to households plus its equivalent nonsurvey data from the Regional Science Research Institute (but using survey-based household and labor information). The RSRI model was aggregated from the 494-sector level to the 52 sectors of the Washington State survey-based model using regional weights and by applying RPCs prior to aggregation. Since according to Table 1, E reproduces well the sectoral ranking of the total linkage difference,³ I deem the measure to be useful. I thus conclude that it “reproduces well” the sectoral rankings of total linkages by examining the rank correlations reported using both Spearman's ρ and Kendall's τ . Pearson's correlation coefficient r is estimated for comparison purposes. Pearson's correlation coefficient measures the correlation of the actual values of the linkage measures, not just the ranks.

The fit of the rankings for the two measures is even better for the top-ranked sectors. Since we should be most interested in surveying such sectors first, these findings reveal even more promise for E .

In the above test, I strictly perturbed the columns of each sector by 30 percent. This amount was somewhat arbitrarily chosen, being the maximum that any sector could be perturbed without having its column in the direct-requirements matrix sum to greater than unity. But what if the 30 percent value was too large? And what if the assumption that rowwise imports estimates are reasonable is incorrect? In answer to the first question, I estimated E five times reducing the column perturbations by 4 percent each time. The rankings were, for all practical purposes, identical to those obtained through the original 30 percent perturbation. By perturbing row coefficients as well as those in columns and by varying each set of coefficients by 5 percent, I did obtain different sectoral rankings, however. The highest Spearman's ρ and Pearson's correlation coefficient (.943 and .816, respectively) were achieved when the row elements of the direct-requirements table were perturbed by 10 percent and the column elements were simultaneously perturbed by 30 percent. Hence, as assumed, the RPCs used were relatively accurate (off by an average of only 10 percent). In fact, when the ranked list of sectors from this table was compared to one that did not vary the row elements at all (only perturbing the column coefficients by 30 percent), no significant difference between them was found (the ρ and

² Elsewhere I (Lahr, 1992) have determined that, for situations such as this, the total linkage measure developed by Meller and Marfàn (1981) is most appropriate. The Ghoshian equivalent or some combination thereof could also be used.

³ The absolute value of the difference between the total earnings linkages produced by the survey-based and nonsurvey tables.

Pearson's correlation coefficients were .987 and .914, respectively). For simplicity, I therefore stuck to using column-only perturbation of 30 percent.

Unsure of which set of weights to use for the tests, I used three: survey-based final demand, nonsurvey output, and employment. Although survey-based final demand data are clearly the most appropriate, they are the least likely to be available to regional analyst. Meanwhile, data on employment would seem unlikely likely to represent well final demand. Nonetheless, they are the data that are most likely to be available at a sufficient level of sectoral detail. Table 1 shows the results of these tests and compares them to the total linkage difference between the survey-based and nonsurvey model weighted by final demand. Surprisingly, employment weights reveal the best results with a Spearman rank correlation coefficient of .75 and Kendall's τ is .57. These reveal significant association between the two sets of data. Moreover, nine of the highest-ranked sectors using E weighted by employment are among the top ten that should be actually targeted for superior data based on absolute total linkage difference. Results using the other two weights fare only marginally less well. Hence, from this example it would appear that nonsurvey tables do a reasonable job in identifying sectors that are most important to target for superior-data collection.

4. A STRATEGY FOR BUILDING A HYBRID REGIONAL INPUT-OUTPUT TABLE

Many of the steps in the procedure that I suggest here are very different from West (1990), who has laid out the best outline to date. First, by following the advice of Stevens and Lahr (1992) aggregation error inherent in West (1990) is avoided. Second, I forward the notion that hybrid model constructors should use the most accurate means available to regionalize and that they use the most accurate technology data available for their region. If a dated detailed survey-based model is available for the region, it should be used as the base model from which superior-data collection strategies should stem. Third, as discussed in the previous section, I assert that sensitivity analysis of sectors to variations in imports both by using and producing industry be performed to identify sectors (i.e., row-column combinations), not cells, that should obtain superior data. Finally, the use of different data sources means varying data reliabilities, the combination of survey data with nonsurvey data is no exception. Consequently, I also assert that formal reconciliation of the various data sources is imperative. The following paragraphs describe a detailed set of procedures that can be used to develop a hybrid regional input-output model.

Step 1, Preparation of Initial Nonsurvey Regional Direct Requirements.

This step comprises the first two in West (1990): developing a technology table and adjusting it to account for regional trade patterns. In developing the technology table, West (1990) lists the following important steps: selecting a base technology table, updating the technology (if necessary), and adjusting the updated table so that international trade is accounted for in a manner that is consistent with the regionalization technique.

The main difference between this step and the two in West (1990) is the regionalization approach. Instead of using the modified location quotient approach described in West (1990), which is perhaps appropriate for the Australian setting in which West and his associates work, I advocate employing a regionalization scheme that allows for cross-hauling, which is the norm in interregional trade rather than the exception.

Step 2, Identifying Sectors for Superior-data Collection.

By omitting aggregation, we can immediately produce a prototype model. The first step in this process is to identify the sectors that should gain superior data. If the technology table is

one “borrowed” from the national input-output table and if the region being modeled comprises only a small proportion of the total economy of the nation, several sets of sectors should be given top consideration for superior-data collection in order to make the most accurate Leontief matrix possible. They are the household-labor sector, resource production sectors (e.g., agriculture, forestry, fishing, and mining) and any aggregate sectors such as those denoted as “miscellaneous” or “not elsewhere classified” or others which, due to regional data problems, are severely aggregated. If funds for data collection are severely limited, however, only one or a few of these sectors will get needed superior data.

If national technology is used, the sectors mentioned in the above paragraph should first be targeted for superior-data collection. But if data-collection funds are severely limited, which of these sectors should be targeted for survey work? Also, if funds are more than sufficient to enable collection of data in all of these sectors or if regional technology is otherwise obtained for these sectors, which sectors should receive superior data using the remaining resources?

Since it has long been established that a prime factor in the instability of regional direct requirements coefficients is regional trade, I have opted to focus on this as a means for identifying sectors for superior data. By using the combined-coefficient sensitivity concept of West (1982)—which has been described in the last section—sectors that make the regional table most sensitive to changes in trade can be easily identified. Since rankings of the sectors using this approach are altered with any coefficient correction (West, 1982), the process of identifying sectors should be performed recursively after the model has been enhanced by each sector’s survey data.

Step 3, Identifying Individual Cells for Data Collection.

To keep data-collection costs for each sector to a minimum and yet maintain accuracy, I have identified (Lahr, 1993) several pieces of information should be obtained for all targeted sectors. These are intermediate inputs and outputs as a proportion of total regional sector output, intrasectoral flows as a proportion of total regional sector output, total regional sector output, and regional labor income. Intermediate input proportions and total regional sector output are necessary to produce regional input proportions, which are essential to proper regionalization (Lahr, 1992). Szyrmer and Lahr (1992) show that intermediate output and total regional output are necessary to estimate properly the RPCs. In addition, Phibbs and Holsman (1981) find that intrasectoral flows generally are one of the larger sets of flows into any given sector. Since Jensen and West (1980) also find that large cells tend to be the most critical to model accuracy, I decided to make the cells on the diagonal of the technology matrix targets for superior data since they tend to be the largest cells in indirect requirements matrices of high order. Finally, it is now well-established (Stevens and Trainer, 1976; Garhart and Giarratani, 1987) that accuracy in the measurement of the household-labor sector is also critical. Hence, if such data are not available at a sufficiently disaggregate areal level from state or local government sources, they should be obtained via survey work.

For targeting cells of a sector for superior data, I used a measure introduced by West (1981). Of course, the measure was applied to every cell in a selected sector’s row and column since a firm should be able to furnish such information. I weighted West’s (1981) single-coefficient change estimates by information using value-added as suggested by Jackson (1991).

I suggest only estimating West’s (1981) measure for a selected sector’s column and row as opposed to all cells of the direct-requirements matrix, as was previously suggested (West, 1990), for algorithm efficiency. The reasoning here is that calculations must be made for each cell. In a typical regional input-output model, this amounts to at least several hundred calculations for a single sector’s row and column.

Step 4, Insertion of Superior Data.

The data for cells identified in Step 3 are inserted into the region's technology table (not the regionalized version, which is typically called the "direct requirements table"). Values for cells that are not survey-based are proportionally adjusted so that each sector's intermediate known input and output levels are met.

Step 5, Biproportional Regionalization.

In order to regionalize using RAS, the intermediate inputs total must sum to intermediate outputs total. Convergence of the RAS technique is achieved when estimated margin totals are within a "reasonable" measure of tolerance.⁴ I suggest that the bases for these tolerances be the relative reliabilities of the corresponding margin data for the row/column. Survey data are the most reliable. Superior data used to estimate intermediate outputs are generally next most reliable. Superior data on intermediate inputs somewhat less reliable. Nonsurvey estimates are considered to be highly unreliable. Using this ranking of reliability, sales and purchases data are reconciled simultaneously with the different data sources.

The process continues by iterating through Steps 2 to 5 until funds for superior-data collection are depleted or the change in the total linkages for each sector has not changed much between consecutive cycles. The matrix that is obtained from the last use of Step 5 is the ultimate matrix to be used for impact or other input-output-based analysis.

For my examples, I set the tolerance limits that terminate the RAS procedure as follows:

- (1) if I used data from the survey-based model, I assigned tolerances of 1 unit (in my case million dollars of shipments) to intermediate inputs and outputs;
- (2) if the sector had no survey-based data and it was a manufacturing sector, I assigned intermediate outputs a tolerance of 30 percent of their initial estimate (the RPC estimates that I used are more accurate for manufacturing sectors);
- (4) if the sector had no survey-based data and it was a manufacturing sector, intermediate inputs were given a tolerance of 100 percent of the initial estimate; and
- (5) if the sector had no survey-based data and it was not a manufacturing sector, I set the tolerance for intermediate inputs and outputs to 100 percent of the initial estimate.

In the case of my examples, I "surveyed" 13 sectors (the number was arbitrarily set at 25 percent of the number of sectors in the model). I tested the approach against the model both open and closed with respect to households. A discussion of the matrix comparison measures—Mean Absolute Deviation (MAD), Weighted Absolute Deviation (WAD), Standardized Total Percentage Error (STPE), r , Root Mean Squared Error (RMSE), and Theil's U —is included in Appendix C.

5. INTERPRETATION OF RESULTS

I hypothesize (1) that error should monotonically decrease as more sectors get superior data and (2) that the percent change in error should approach zero as more sectors get superior data. That is, the two hypotheses are (1) each additional sector "surveyed" reduces error and (2) there are decreasing marginal returns to accuracy from superior data.

⁴ In the RAS algorithm, which I used, rows and columns of the direct requirements matrix are adjusted sequentially. After these adjustments, new values for intermediate inputs and outputs are then estimated. These new values are next compared to the actual values. The algorithm continues with such row and column adjustments until the estimated values are sufficiently close to the known ones or until a pre-specified number of row and column adjustments is exceeded. The measure of "sufficiently close" is the tolerance level that must be specified by the user of the algorithm.

Figures 1 to 3 show test results when the model is closed with respect to households and Figures 4 to 6 show the open-model results. It appears that the first hypothesis holds: error is reduced by additional partial survey work. This hypothesis is violated most in technology matrices (Figures 1 and 4) and second most in Leontief inverses (Figures 2 and 5). In these cases, by examining only the WAD measure, which weighs error in larger coefficients disproportionately (Jensen and West, 1980), not only are fewer violations observed but also those that still exist are less severe. The WAD measure is not as appropriate as some of the other measures for examining error in multipliers (Figures 3 and 6), because no rationale exists for specifying that results for larger sectors need to have lower proportions of error than those for smaller sectors. Hence, the WAD measure for multipliers can be discounted in the case of multipliers but have been included in the figures for the sake of completeness.⁵

The second hypothesis (diminishing marginal returns to superior data) appears to hold well for the model closed to households (Figures 1 to 3). That these figures reveal a vacillating diminishing trend to the change in error is no surprise; there may be little difference between the “survey” data that is inserted and the nonsurvey data it is replacing. The results shown in Figures 4 and 5 are somewhat disappointing, however. These figures reveal an early five-sector period (the third through seventh sectors chosen) with little or no decrease in hybrid model error. It is disappointing to learn that the funds needed to obtain such data could yield so little partitive accuracy.

The results for open-model multipliers (Figure 6) are more favorable with error change consistently less than zero. Regardless of the model type, however, “surveys” for only four sectors were required to reduce error to 50 percent (an STPE error of 24 to about 11 in the closed model) of its original value. For the closed model these sectors were: FIRE, Trade, Services, and Construction; all of which are very aggregate sectors. Aluminum replaced Construction as a major error-reducing sector in the open model.

How do these results compare to the case where, in each successive round of survey work, the sector with the greatest absolute deviation in total linkages between the closed survey-based and nonsurvey models is known? Figures 7 through 9 reveal that, in general, similar results are obtained. The absolute amount of error remaining for each successive sector surveyed is constantly less for all measures than it was in the for the measures supporting Figures 1 through 3, however. Surveys of only two sectors—FIRE and Construction—were required to reduce error for multipliers to about 36 percent of its original value (to an STPE error of about 5.5). The two late downward spikes in Figures 7 and 8 are from technology changes in Dairy Products and Logging. The trade adjustments in Aluminum account for the largest downward spike for output multipliers in Table 2 is a comparison of the open- and closed-model sectors that my approach suggests should be “surveyed” (not including the household labor sector in the 52-sector model, which is the top-ranked sector). These should be compared with the sectors that would be modeled if the recursive superior-data collection technique had been employed with perfect information on the differences between the survey-based and nonsurvey models (see Table 2, bottom). In both cases, the nonsurvey model identifies 7 of the top 13 sectors to be targeted. On the other hand, several critical sectors not identified by my approach are resource-based sectors: Fisheries, Mining, Vegetables and Fruits, and Forestry. I earlier made the point that such sectors should automatically be considered for receiving superior data since they have technology that is likely to be very different from the national average technology represented in

⁵ Note that errors for multipliers are estimated by subtracting unity from the value of the multiplier. This is done because when measuring the accuracy of tables, the relative accuracy direct effects should *not* be included: they are should be estimated outside of the model.

the nonsurvey model. I chose not to do so because of the severe aggregation in the survey-based Washington State model, which would have left few sectors to test the approach presented here.

6. CONCLUSIONS

It appears that the algorithm developed in this paper is successful in drastically improving both the partitive and holistic accuracy of nonsurvey tables. In particular, there was evidence vindicating a priority to the collection of superior data for the household/labor sector, resource production sectors, and any aggregate goods-producing sectors denoted as “miscellaneous” or “not elsewhere classified.” Perhaps because of loose tolerances for sectors that did not get superior data in the RAS procedure, it turns out the hypotheses that (1) each additional sector “surveyed” reduces error and (2) there are decreasing marginal returns to superior data are not found to be *strictly* true. Nonetheless, the research presented in this paper *provides evidence of a general tendency* supporting both of these hypotheses. For example, error did tend to decrease as more sectors were “surveyed,” and there were general improvements in the various components of the tables as the process proceeded. Hence, there is hope that similar approaches, (e.g., alternatives with more stringent RAS tolerances) might favor the hypotheses. In any case, more testing must be performed to validate variations of the procedure outlined in this paper.

The strategy and the interpretation of the paper’s findings do lend themselves to a wider set of applications than have been intimated. For example, although these experiments were conducted on a sector-by-sector basis, one certainly can consider each sector, in the experiments above, to be a group of sectors. In this way, my suggestion that sectors should be surveyed sequentially becomes one of surveying sets of sectors in successive waves. Further, there is no reason that this procedure could not be applied to updating a national model as opposed to regionalizing one. I leave speculation and investigation of other possible applications of the strategy discussed in this paper to others.

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TABLE 1: Rankings of Sectors for Survey Work: Actual Versus Estimates

| Industry # | Total Linkage Difference | | West's (1982) Sensitivity by Weight Type | | | | | |
|------------|--------------------------|------|--|--------|---------|--------|------------|--------|
| | Measure | Rank | Final Demand | | Output | | Employment | |
| | | | Measure | Rank | Measure | Rank | Measure | Rank |
| 1. | 323 | 9 | 290 | 10 | 1,107 | 8 | 188 | 11 |
| 2. | 124 | 22 | 239 | 14 | 291 | 19 | 263 | 7 |
| 3. | 163 | 17 | 264 | 11 | 1,246 | 7 | 219 | 10 |
| 4. | 15 | 44 | 33 | 43 | 53 | 40 | 26 | 36 |
| 5. | 72 | 34 | 35 | 41 | 143 | 30 | 24 | 38 |
| 6. | 155 | 18 | 178 | 20 | 562 | 12 | 117 | 16 |
| 7. | 66 | 36 | 132 | 25 | 383 | 15 | 88 | 19 |
| 8. | 128 | 21 | 224 | 17 | 300 | 18 | 78 | 22 |
| 9. | 14 | 45 | 74 | 33 | 161 | 28 | 24 | 39 |
| 10. | 13 | 46 | 101 | 29 | 129 | 32 | 39 | 31 |
| 11. | 57 | 37 | 150 | 22 | 341 | 16 | 84 | 20 |
| 12. | 12 | 47 | 7 | 51 | 8 | 52 | 3 | 52 |
| 13. | 48 | 39 | 50 | 37 | 42 | 43 | 21 | 43 |
| 14. | 129 | 20 | 57 | 36 | 183 | 25 | 39 | 32 |
| 15. | 220 | 13 | 19 | 49 | 82 | 35 | 18 | 45 |
| 16. | 143 | 19 | 263 | 12 | 506 | 13 | 121 | 15 |
| 17. | 113 | 25 | 505 | 9 | 586 | 11 | 163 | 13 |
| 18. | 5 | 49 | 225 | 16 | 172 | 27 | 71 | 24 |
| 19. | 50 | 38 | 141 | 24 | 138 | 31 | 43 | 29 |
| 20. | 15 | 43 | 30 | 45 | 56 | 37 | 22 | 41 |
| 21. | 112 | 26 | 141 | 23 | 192 | 24 | 38 | 33 |
| 22. | 78 | 33 | 181 | 19 | 209 | 22 | 66 | 25 |
| 23. | 114 | 24 | 173 | 21 | 158 | 29 | 52 | 28 |
| 24. | 243 | 12 | 73 | 34 | 217 | 21 | 99 | 17 |
| 25. | 78 | 32 | 124 | 27 | 181 | 26 | 53 | 26 |
| 26. | 29 | 41 | 8 | 50 | 26 | 46 | 7 | 50 |
| 27. | 177 | 14 | 130 | 26 | 203 | 23 | 52 | 27 |
| 28. | 33 | 40 | 4 | 52 | 10 | 51 | 5 | 51 |
| 29. | 90 | 28 | 108 | 28 | 324 | 17 | 76 | 23 |
| 30. | 117 | 23 | 24 | 47 | 37 | 44 | 21 | 42 |
| 31. | 11 | 48 | 32 | 44 | 113 | 34 | 16 | 48 |
| 32. | 567 | 7 | 562 | 8 | 454 | 14 | 97 | 18 |
| 33. | 168 | 16 | 38 | 39 | 33 | 45 | 26 | 35 |
| 34. | 66 | 35 | 33 | 42 | 54 | 39 | 17 | 46 |
| 35. | 2 | 51 | 35 | 40 | 12 | 50 | 14 | 49 |
| 36. | 90 | 29 | 23 | 48 | 18 | 48 | 24 | 40 |
| 37. | 1 | 52 | 62 | 35 | 25 | 47 | 27 | 34 |
| 38. | 2 | 50 | 42 | 38 | 17 | 49 | 16 | 47 |
| 39. | 549 | 8 | 734 | 7 | 128 | 33 | 240 | 9 |
| 40. | 82 | 31 | 100 | 30 | 44 | 42 | 25 | 37 |
| 41. | 112 | 27 | 225 | 15 | 51 | 41 | 144 | 14 |
| 42. | 22 | 42 | 82 | 32 | 55 | 38 | 39 | 30 |
| 43. | 1,416 | 6 | 1,311 | 6 | 2,536 | 6 | 1,085 | 6 |
| 44. | 273 | 11 | 221 | 18 | 678 | 10 | 168 | 12 |
| 45. | 85 | 30 | 26 | 46 | 79 | 36 | 19 | 44 |
| 46. | 173 | 15 | 92 | 31 | 278 | 20 | 80 | 21 |
| 47. | 294 | 10 | 248 | 13 | 747 | 9 | 240 | 8 |
| 48. | 3,389 | 3 | 4,399 | 2 | 6,017 | 5 | 1,939 | 5 |
| 49. | 2,963 | 4 | 3,406 | 3 | 8,006 | 3 | 3,202 | 2 |
| 50. | 3,905 | 2 | 2,768 | 4 | 9,903 | 2 | 2,526 | 4 |
| 51. | 2,027 | 5 | 2,614 | 5 | 7,989 | 4 | 3,145 | 3 |
| 52. | 4,634 | 1 | 9,399 | 1 | 25,245 | 1 | 6,249 | 1 |
| ρ | | | .6538 | (7.9) | .7237 | (9.7) | .7487 | (10.6) |
| τ | | | .4949 | (27.0) | .5477 | (29.9) | .5688 | (39.0) |
| r | | | .9293 | | .9244 | | .9052 | |

Notes: Numbers in parentheses after Spearman's ρ is its t statistic and that after Kendall's τ is its z score. Both test the hypothesis that there is no relationship between the subject measure and the linkage difference measure. The linkage difference measure is the absolute difference between the total linkages for the survey-based and nonsurvey models, with total linkages as in Meller and Marfán (1981).

Industries are defined in Appendix B.

TABLE 2: Sectors Targeted for Survey Work

The Recursive Search Technique

Open Model

1. Construction
2. FIRE
3. Trade
4. Services
5. Aluminum
6. Sawmills
7. Transportation Services
8. Meat Products
9. Livestock and Products
10. Aerospace
11. Logging
12. Canning and Preserving
13. Paperboard and Other Paper Products

Closed Model

- FIRE
- Trade
- Services
- Construction
- Transportation Services
- Aerospace
- Meat Products
- Communications
- Livestock and Products
- Sawmills
- Logging
- Field and Seed Crops
- Aluminum

Maximum Absolute Deviation in Total Linkages

Closed Model

1. FIRE
2. Construction
3. Fisheries
4. Trade
5. Livestock and Products
6. Aluminum
7. Mining
8. Vegetables and Fruits
9. Services
10. Petroleum
11. Dairy Products
12. Forestry
13. Logging

APPENDIX A

STRUCTURAL MATRIX ERRORS IN INTERINDUSTRY RELATIONS ESTIMATES: A DERIVATION OF EVANS'S (1954) FORMULA

Using \mathbf{A} and \mathbf{I} to represent the n -order direct-requirements and identity matrices, respectively, consider a new matrix that is formed by a perturbation of \mathbf{A} that is expressed as $\mathbf{A}^* = \mathbf{A} - \mathbf{P}$, where $\mathbf{P} = [p_{ij}a_{ij}]$. Then,

$$(\mathbf{I} - \mathbf{A}^*) = (\mathbf{I} - \mathbf{A}) + \mathbf{P}$$

which when letting $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ and $\mathbf{B}^* = (\mathbf{I} - \mathbf{A}^*)^{-1}$ leads to either of the following:

$$\mathbf{B}^* = \mathbf{B}(\mathbf{I} - \mathbf{PB})^{-1}$$

$$= (\mathbf{I} - \mathbf{BP})^{-1} \mathbf{B}$$

Subsequently, the matrix of errors produced in a Leontief inverse, $\mathbf{E} = \mathbf{B}^* - \mathbf{B}$, by a perturbation in a direct-requirements matrix is

$$\mathbf{E} = -\mathbf{B}[\mathbf{I} - (\mathbf{I} - \mathbf{PB})^{-1}]$$

$$= [(\mathbf{I} - \mathbf{BP})^{-1} - \mathbf{I}]\mathbf{B}$$

This is equivalent to that derived by West (1982), who showed that

$$\mathbf{E} = (\mathbf{I} - \mathbf{BP})^{-1} \mathbf{BPB}$$

and that by Dwyer and Waugh (1953), who showed that

$$\mathbf{E} = -\mathbf{BPB}(\mathbf{I} + \mathbf{PB})^{-1}$$

That there are two fewer matrix multiplications to be performed makes the Evans's formula more practical.

APPENDIX B

INDUSTRIES IN THE 1972 WASHINGTON INPUT-OUTPUT STUDY

| # Industry Name | U.S. Standard Industrial Classification Code(s) |
|-----------------------------------|---|
| 1. Field and Seed Crops | 011, 013 (except 0133), pt. 019 |
| 2. Vegetables and Fruits | 0133, 016, 017, pt. 019 |
| 3. Livestock and Products | 02 (except 027), 075 |
| 4. Other Agriculture | 018, 027, 07 (except 074, 075) |
| 5. Fisheries | 09 (except 097) |
| 6. Meat Products | 201 |
| 7. Dairy Products | 202 |
| 8. Canning and Preserving | 203, 2091, 2092 |
| 9. Grain Mill Products | 204 |
| 10. Beverages | 208 |
| 11. Other Foods | 205-207, 2095-2099 |
| 12. Textiles | 22 |
| 13. Apparel | 23 |
| 14. Mining | 10-14 |
| 15. Forestry | 08, includes national and state forests |
| 16. Logging | 241 |
| 17. Sawmills | 242 |
| 18. Plywood | 2435, 2436 |
| 19. Other Wood Products | 2431, 2434, 2439, 244-249 |
| 20. Furniture and Fixtures | 25 |
| 21. Pulp Mills | 261 |
| 22. Paper Mills | 262 |
| 23. Paperboard & Oth. Paper Prod. | 263-266 |
| 24. Printing and Publishing | 27 |
| 25. Industrial Chemicals | 281, 286-289 |
| 26. Other Chemicals | 282-285 |
| 27. Petroleum | 29 |
| 28. Glass Products | 321-323 |
| 29. Cement, Stone, and Clay | 324-329 |
| 30. Iron and Steel | 331, 332, 3398, 3399 |
| 31. Other Nonferrous Metals | All other 33 |
| 32. Aluminum | 3334, 3353-3355, 3361 |
| 33. Structural Metal Products | 344 |
| 34. Other Fabricated Metals | 34 (except 344) |
| 35. Nonelectrical Motive Equip. | 351-353 |
| 36. Machine Tools and Shops | 354, 359 |
| 37. Nonelectrical Ind. Equip. | 355-358 |
| 38. Electrical Machinery | 36 |
| 39. Aerospace | 372, 276 |
| 40. Motor Vehicles | 371, 374, 375, 379 |

| | |
|-----------------------------|--|
| 41. Ship and Boat Building | 373, incl. Puget Sound Naval shipyard |
| 42. Other Manufacturing | 30, 31, 38, 39 |
| 43. Transportation Services | 40-47, inc. Post. Serv. & public transportation |
| 44. Electric Companies | 491, pt. 493, incl. government enterprises |
| 45. Gas Companies | 492, pt. 493, incl. government enterprises |
| 46. Other Utilities | pt. 493, 494-497, includes government enterprises. |
| 47. Communications | 48 |
| 48. Construction | 15-17 |
| 49. Trade | 50-59, includes state liquor stores |
| 50. FIRE | 60-67 |
| 51. Services | 074, 097, 70-89 (excl. publicly owned schools & hospitals) |
| 52. Households/Labor | |

APPENDIX C

SOME MEASURES FOR COMPARING INPUT-OUTPUT MATRICES

C1. INTRODUCTION

Despite extensive evaluation of regional input-output tables derived via the various nonsurvey methods, there is still some question regarding the relative accuracy of the various techniques when compared to survey-based tables (Miernyk, 1976, p.54). In part, this is due to the age-old problem in regional analysis—the lack of proper and sufficient survey data to which estimates can be compared (Polenske, 1989; Miernyk, 1976, p. 54). In addition, even provided proper and sufficient “superior” data, comparisons are generally made using measures whose properties have not been investigated. That is, there is no proven strict relationship between the value of the measures and the amount of error inherent in an estimate, especially in the case of the application of input-output tables. Thus, the results of the comparisons have been, at best, inconclusive with regard to the goodness-of-fit of the estimates.

Measures of distance and association serve two purposes, (1) to measure the ability of models to produce accurate results and (2) to determine the statistical significance of the difference between the actual and estimated data (Knudsen and Fotheringham, 1986) [henceforth, K-F]. Unfortunately, other than K-F few researches have ever thoroughly studied measures of distance or association for matrix comparison problems. And, certainly, no one else has considered and compared a battery of such measures in a scientific way.

In total, Lahr (1992) found 14 different measures that have been used to determine the accuracy of I-O models. The reason so many have been used is that the tables, themselves, are unlike those of other matrix-based economic models. The structure of I-O tables and the possible uses toward which they can be applied require a very stringent set of properties for a general-use comparison measure. Hence, researchers have chosen a measure they have used in other statistical applications or have used measures used by other I-O analysts, despite their possible impropriety.

In the literature, at most 6 of the 14 measures have been discussed and/or tested with regard to the validity of a few of their properties. Not even all desired traits have been discussed in these few measures. In fact, testing of the measures was only evident in one paper, Miller and Blair (1982). Rather those measures not perceived as being well-suited to a particular application are either altered slightly (creating a new measure) or are omitted from use. Hence, despite passage of over two decades, it is difficult to disagree with Miernyk (1975, p.34) “that there is some ambiguity in the measurement of projection error” and that “there is an opportunity for an innovative statistician to make an important contribution here.” I suspect, however, that it also may take work along the lines of that by Asami and Smith (1995), who used an axiomatic approach to identify appropriate measures.

C2. NOTATION

At this point, I set out the notation that will be used throughout this appendix. Following the notation of Miller and Blair (1985), a_{ij} is the i th element down the j th column of matrix A of dimension ($m \times n$). The technology matrix, \mathbf{A} , has an estimate $\hat{\mathbf{A}}$ with elements designated \hat{a}_{ij} .

Likewise, μ and $\hat{\mu}$ are the vectors of actual and estimated multipliers, and μ_i and $\hat{\mu}_i$ are the respective multipliers of a specific sector i . Throughout I have not separately designated the formulae for transactions or total requirements matrices which, besides mere variable-notation changes, have formulae identical to those of the technology matrix.

C3. THE ERROR MEASURES

Standardized Total Percent Error

Of the measures used in this paper, the first to be used in I-O applications was the weighted average percentage error measure (Leontief, 1966, p. 244). Somehow Leontief's use of the measure was forgotten, because it was not used again until Sawyer and Miller (1983), where it was apparently reinvented as the mean absolute deviation as a percentage of the mean coefficient (MPMC), and Miller and Blair (1982, 1983) where it was given its most commonly used, current name—standard total percentage error (STPE). In an unpublished, paper Szyrmer (1984) used this measure under yet another name, mean normalized deviation. It has the formula

$$100 \frac{\sum_j \sum_i |a_{ij} - \overset{\circ}{a}_{ij}|}{\sum_j \sum_i a_{ij}}$$

Its only major drawback is that it may not be exceptionally sensitive to high-valued cells.

Correlation Coefficient (r)

The correlation coefficient, r , isolated from the rest of the regression parameters and statistics has been used as a measure of association. To make it a distance measure, Szyrmer (1984) transformed it into what he called the disparity index, defining this “new” measure as $50(1-r)$. Hence, at perfect correlation its value is 0, at no correlation its value is 50 and at perfect negative correlation its value is 100. To keep the direction of the values of r in line with those of the other measures, I have merely subtracted its value from 100.

Mean Absolute Difference

Smith and Morrison (1974) and Morrison and Smith (1974) introduced the use of the mean absolute difference (MAD) measure to I-O analysis. Its formula is

$$100 \frac{\sum_j \sum_i |a_{ij} - \overset{\circ}{a}_{ij}|}{mn}$$

As with many other measures, there is no penalty in this measure for having error in high-valued coefficients as opposed to low-valued ones. In addition, its magnitude changes with the order (size) of the I-O tables being evaluated. In answer to the first of these two problems, I developed the weighted absolute difference (WAD), which is discussed later.

Index of Inequality (Theil's U)

A measure originally developed in Theil et al. (1966) was first used in the input-output literature by Stevens and Trainer (1976). This measure, commonly called Theil's index of inequality (U), can be broadly interpreted as a standardized root mean squared error. (The root mean squared error measure is discussed later in this appendix.) If multiplied by 100 percent has an interpretation that is very similar to that of the STPE measure. It has the advantage of yielding

an overall distance proportion as well as three other proportions: bias (U_M), variance (U_V), and covariance (U_C). When error is attributed to bias, it means the estimated values tends to be consistently higher or lower than the actual values. When it is attributed to variance, then there is an apparent pattern in the error structure that is correlated with the actual values. These extra error proportions can, therefore, provide valuable insight to the researcher who is seeking better estimation procedures. The formula for index of inequality is

$$\left[\frac{\sum_j \sum_i (a_{ij} - \hat{a}_{ij})^2}{\sum_j \sum_i a_{ij}^2} \right]^5$$

This measure is probably used to its full potential in a paper by Stevens, Treyz, and Lahr (1989), where the authors examine the accuracy of various RPC-estimating techniques. Stevens, Treyz and Lahr, however, do not look at any of the I-O tables or their components that are being analyzed in this study. Its main drawback is that, as with most of the other measures, it does not penalize more heavily for error in higher-valued coefficients.

Root Mean Squared Error (RSME)

The Root Mean Squared Error or Euclidean Metric Distance is defined as

$$\frac{\left[\sum \sum (a_{ij} - \hat{a}_{ij})^2 \right]^5}{mn}$$

Harrigan, McGilvray, and McNicoll (1980) introduced this well-known measure to the I-O literature. As with others such as MAD, this measure does not yield any idea of the relative difference between two matrices, but rather only the average total difference. Hence, one cannot really determine how bad or good an estimated matrix is when compared to the actual. To correct for the latter problem, Theil et al. (1966) developed the index of inequality, which was discussed earlier.

Weighted Absolute Difference (WAD)

To my knowledge Lahr (1992) is the first to use the WAD. It is designed to make up for the problems of most of the other measures. Its formula is

$$\frac{\sum_j \sum_i (a_{ij} + \hat{a}_{ij}) \cdot |a_{ij} - \hat{a}_{ij}|}{\sum_j \sum_i (a_{ij} + \hat{a}_{ij})}$$

That is, the $(a_{ij} + \hat{a}_{ij})$ -term weights the absolute difference term so that the errors of large cells are emphasized. In this way the measure is extremely sensitive to error in large cells, something that, so Jensen and West (1980) have told us, is critical. The advantage of this measure is that if either of the matrices is nonzero for a cell, the measure's value is not undefined. Clearly, the $(a_{ij} + \hat{a}_{ij})$ -term can become a problem if the one of the two matrices is survey-based since the weighting process applies the size of the elements in both matrices equally. In such cases only a_{ij} should be used as the weighting term, and then error is measured as zero whenever the actual

matrix takes the value zero, which can be problematic. There is one additional problem with this measure, it does not generally express proportional error well as do STPE or Theil's U.

FIGURE 1: Error Change for Sequence Surveyed—Model Closed to Households
Direct Requirements

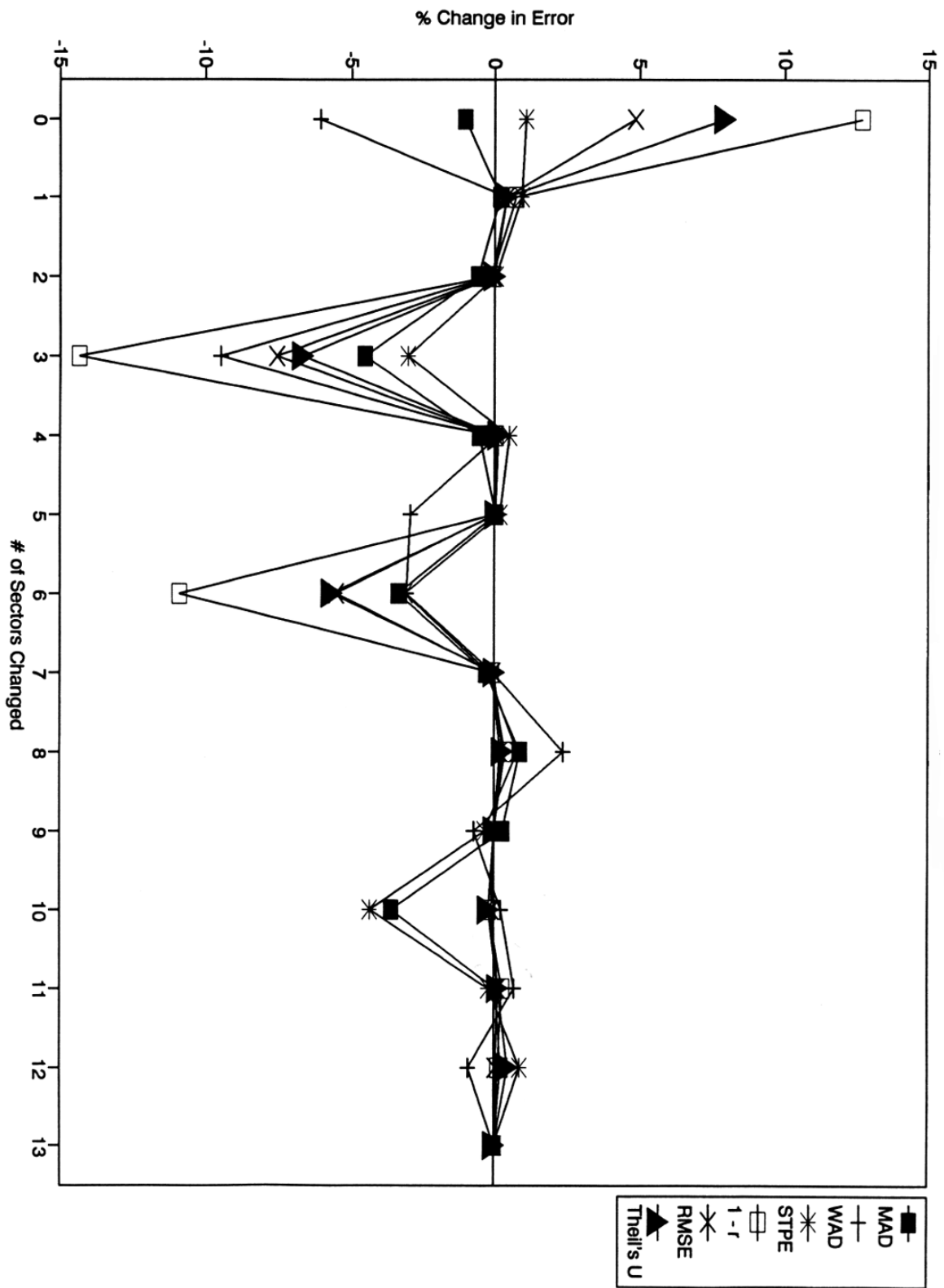


FIGURE 2: Error Change for Sequence of Sectors Surveyed—Model Closed to Households
The Leontief Inverse

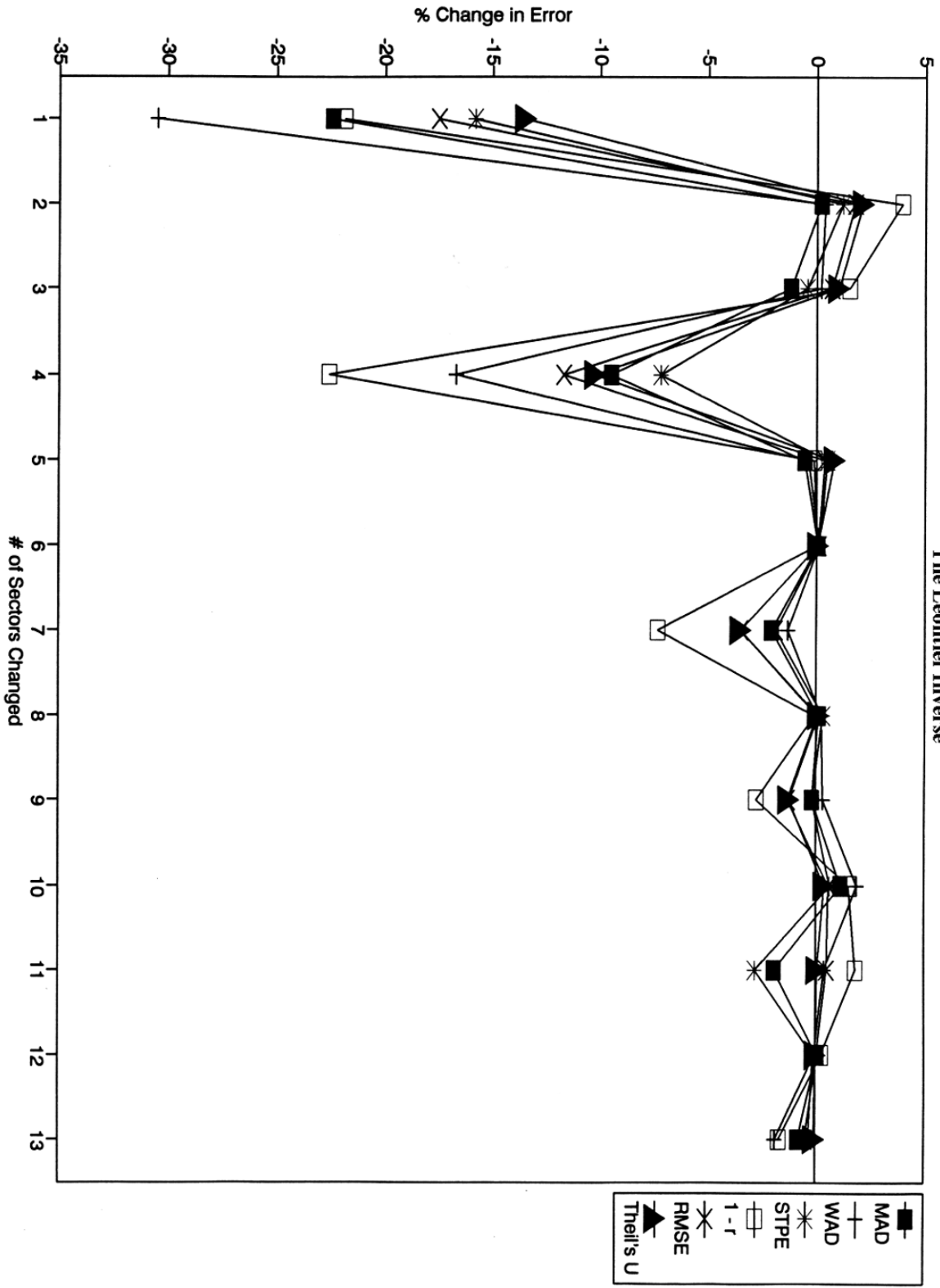
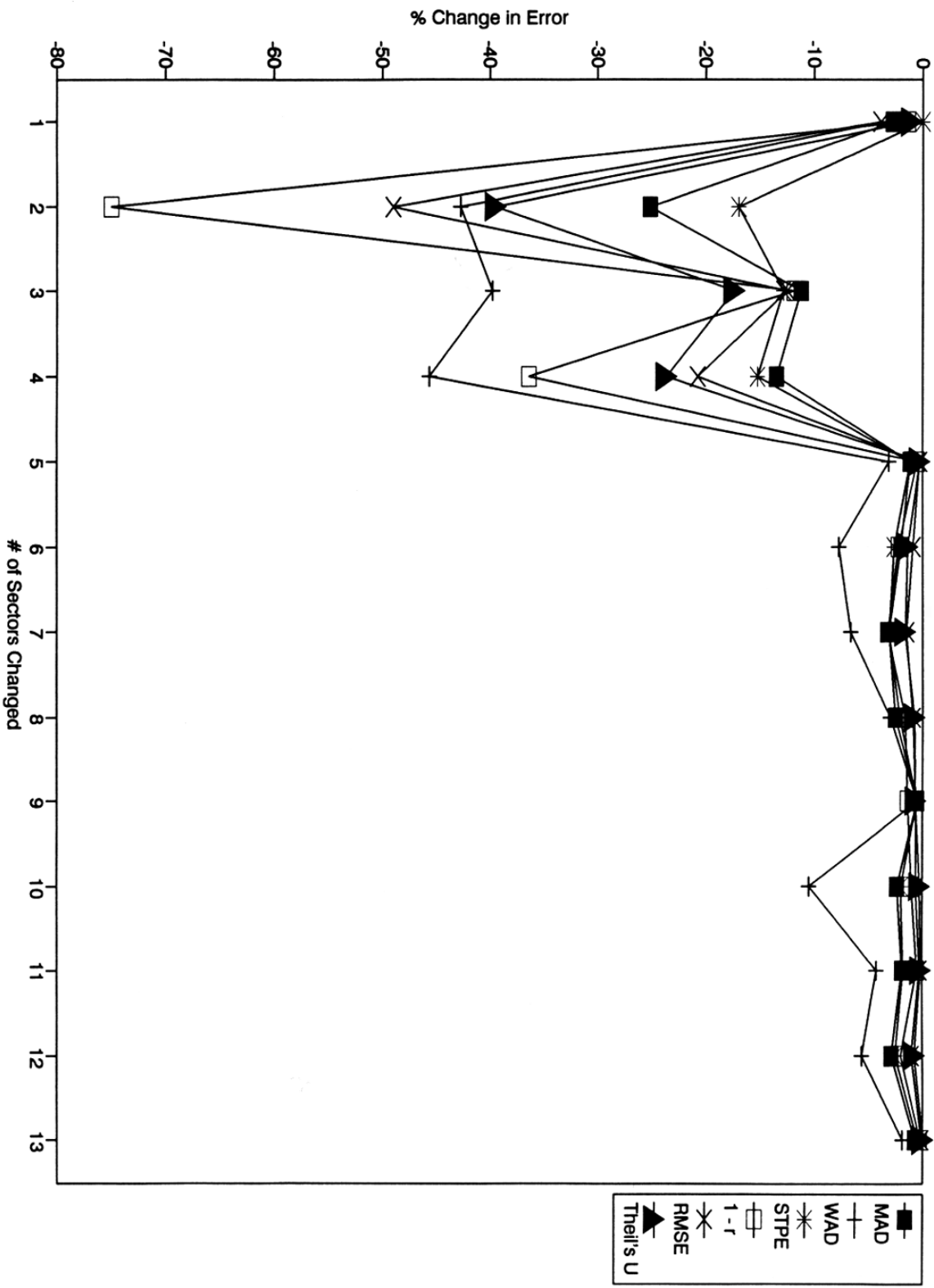


FIGURE 3: Error Change for Sequence of Sectors Surveyed—Model Closed to Households
Output Multipliers



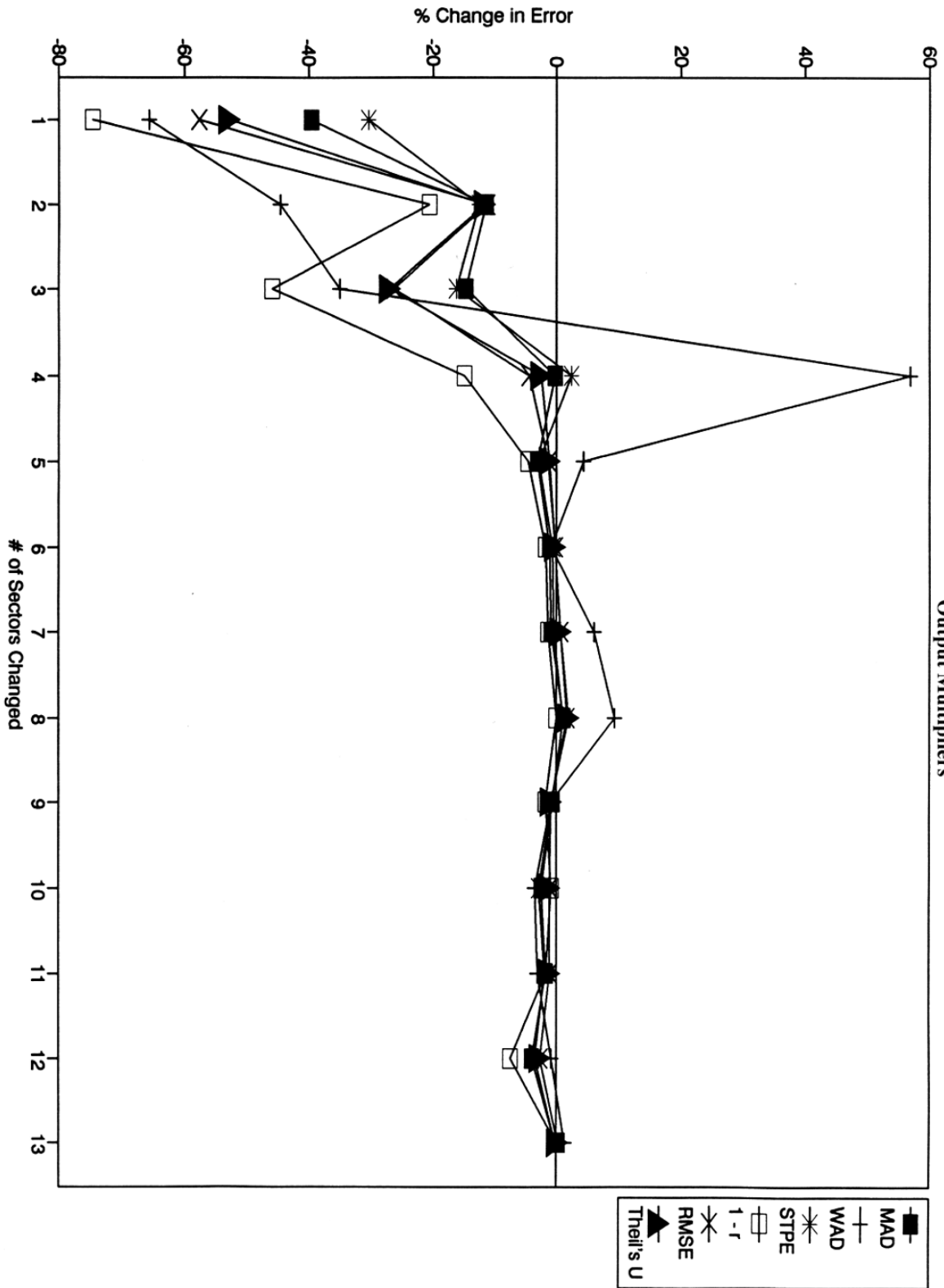


FIGURE 5: Error Change for Sequence of Sectors Surveyed—Model Open to Households
Direct Requirements

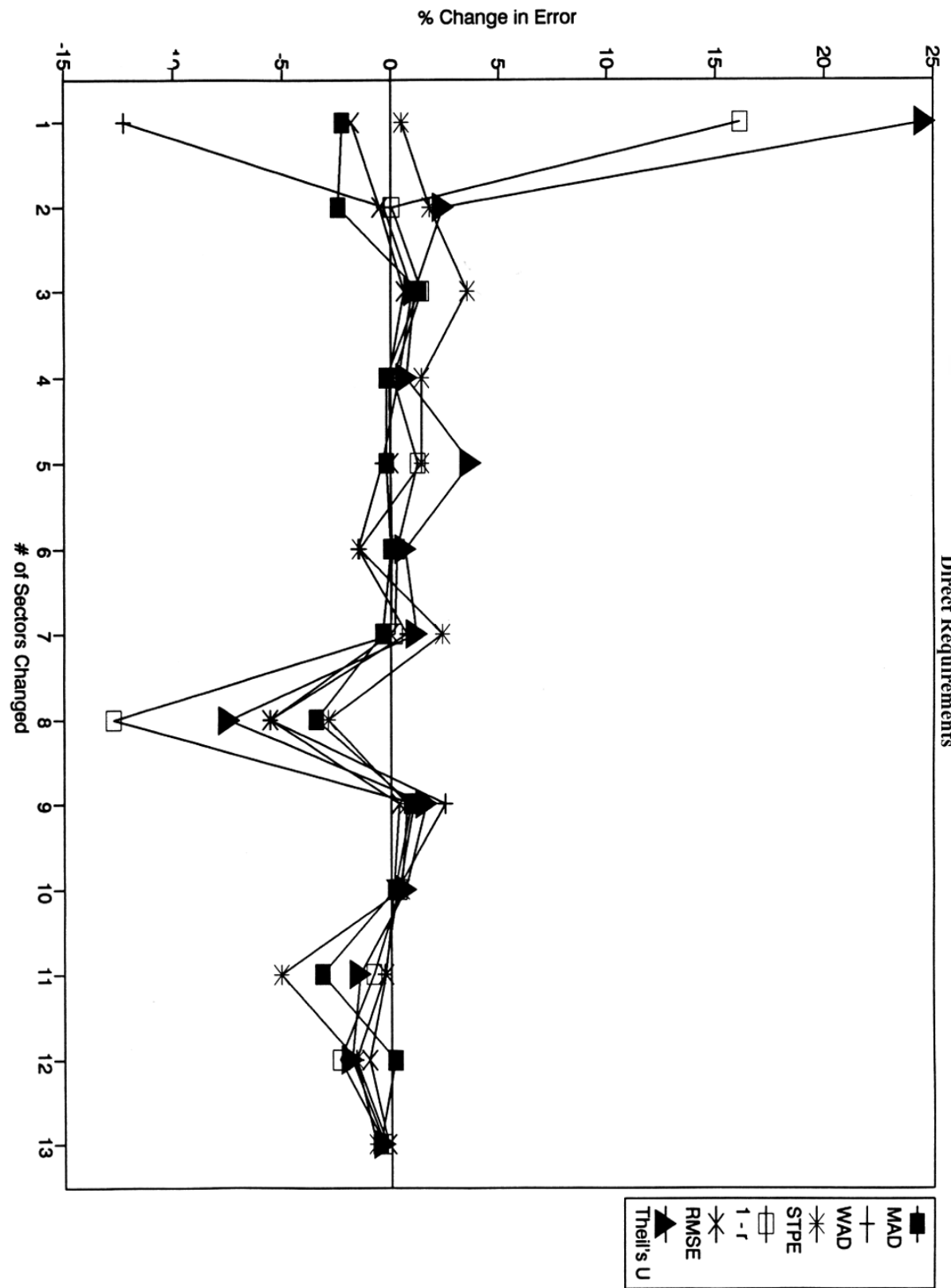
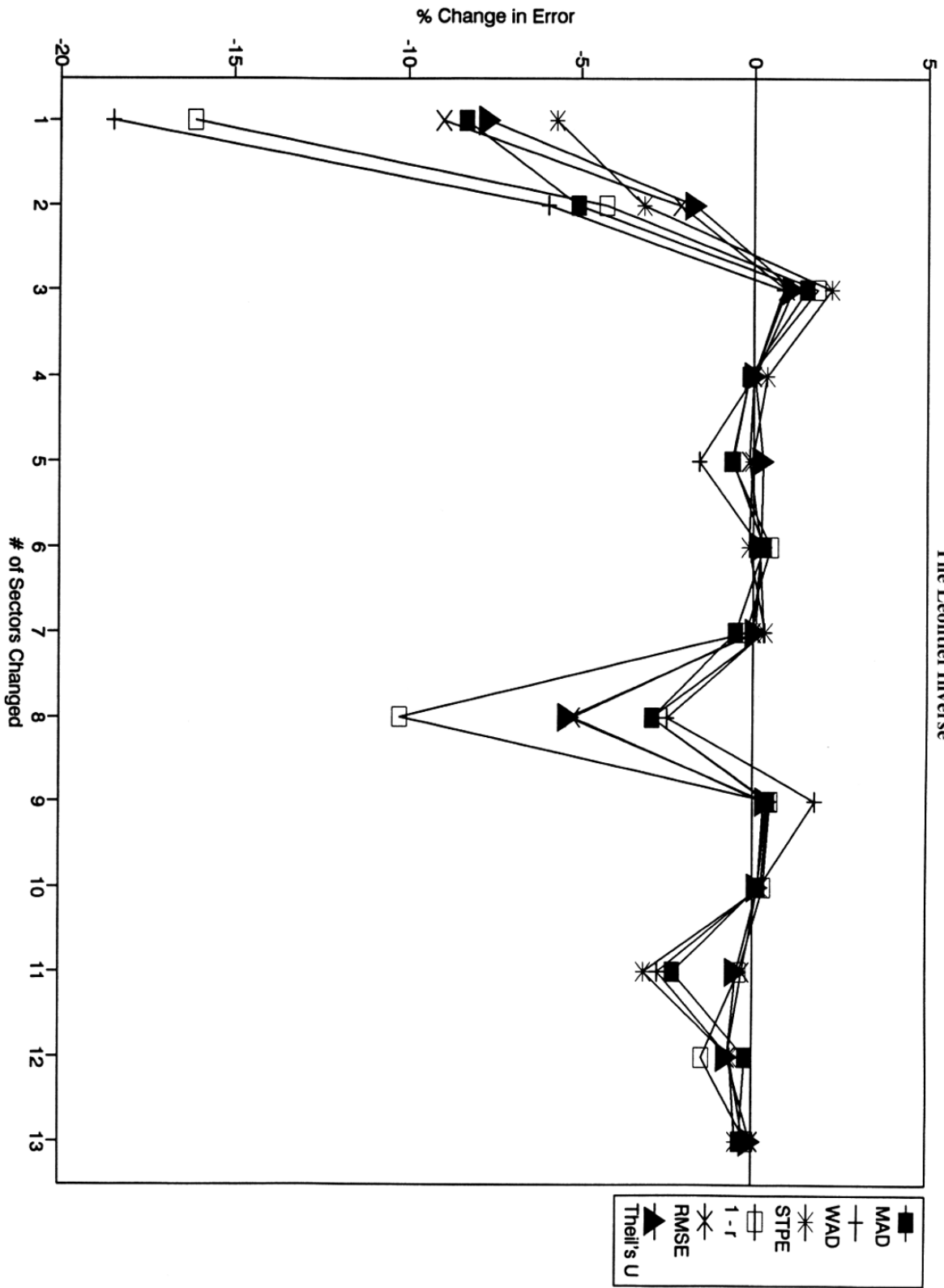


FIGURE 6: Error Change for Sequence of Sectors Surveyed—Model Open to Households
The Leontief Inverse



**FIGURE 7: Error Change for Sequence of Sectors Surveyed—Total Linkage Difference Closed
Direct Requirements**

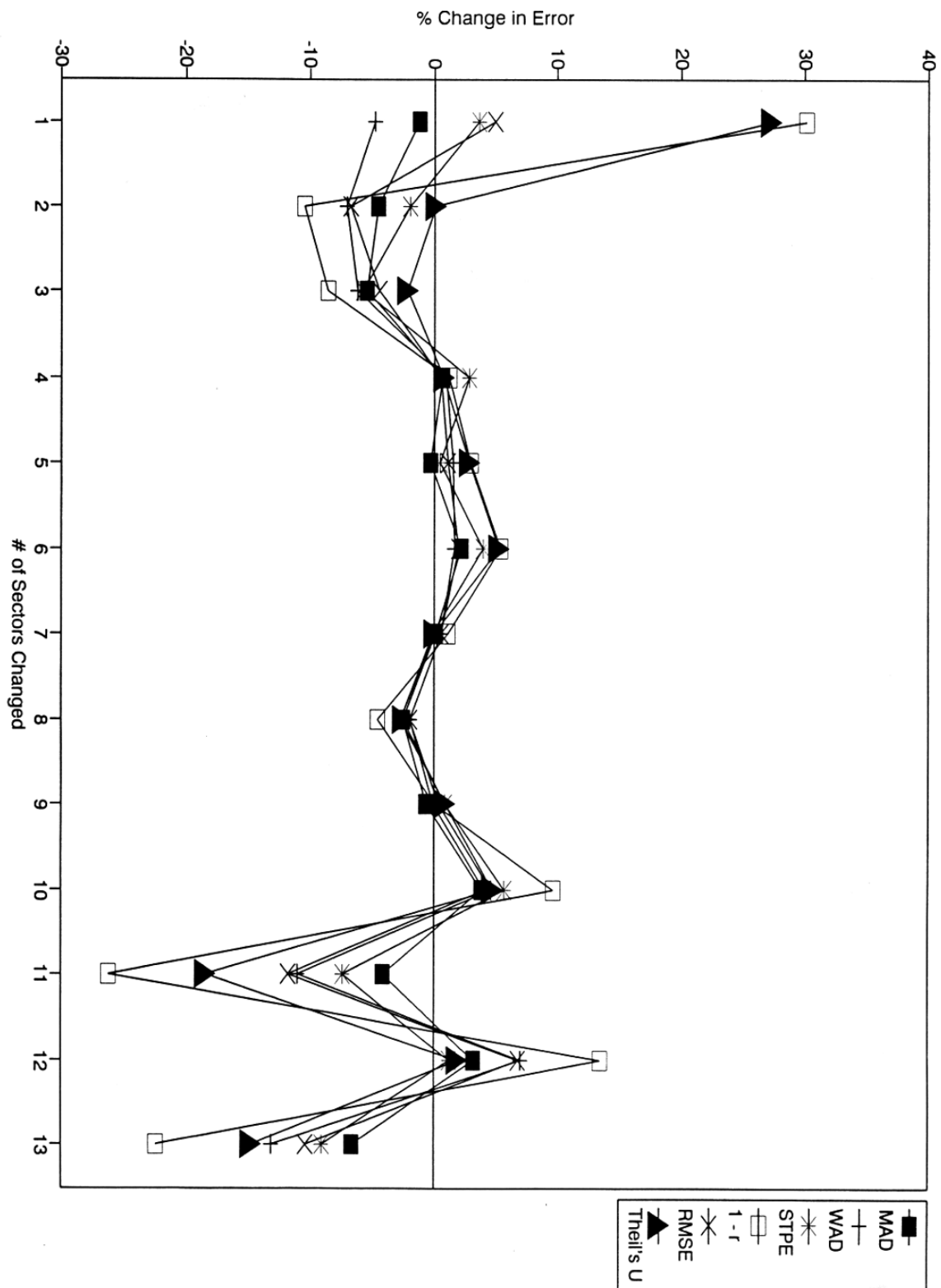
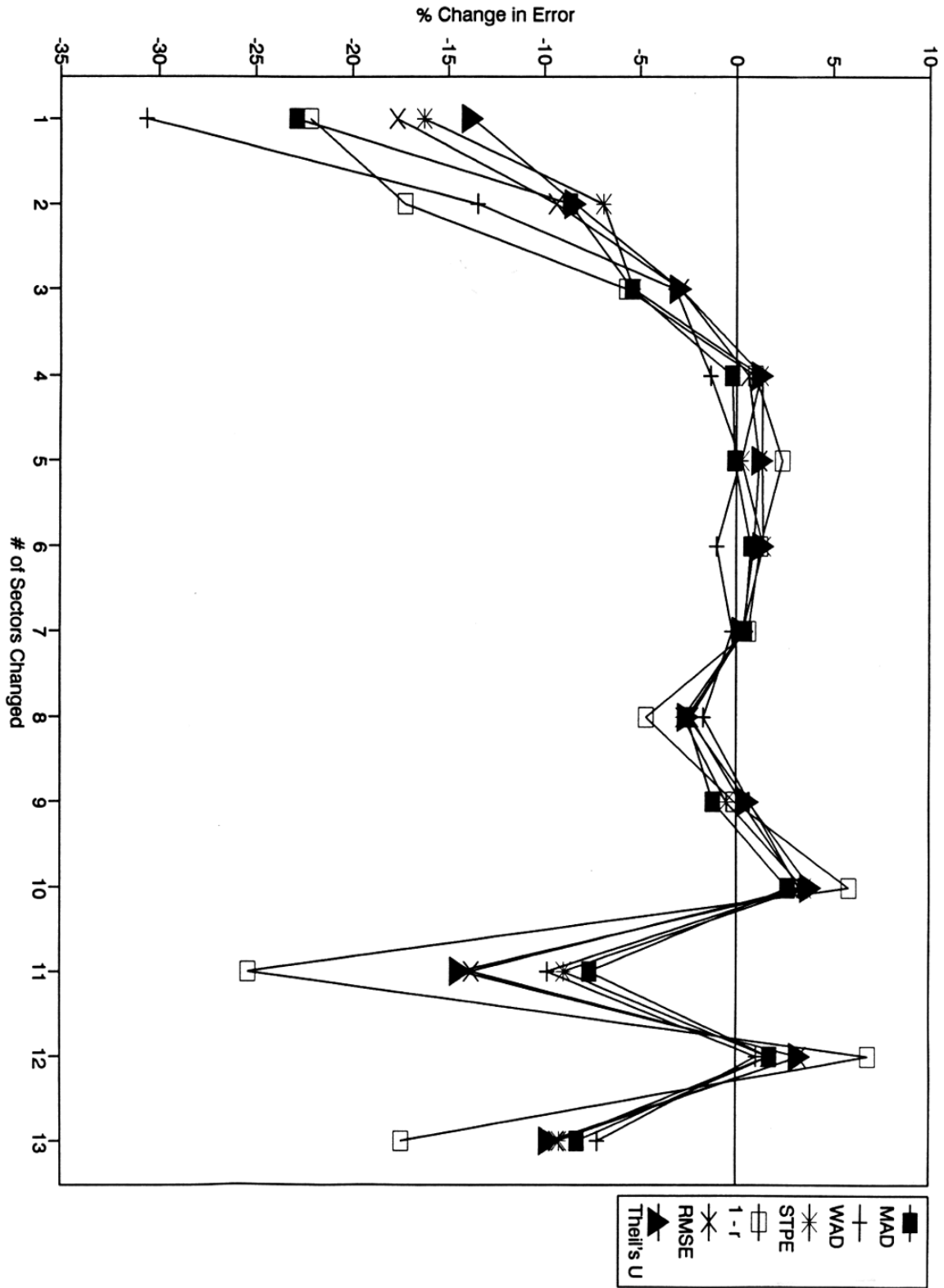


FIGURE 8: Error Change for Sequence of Sectors Surveyed—Total Linkage Difference Closed
The Leontief Inverse



**FIGURE 9: Error Change for Sequence of Sectors Surveyed—Total Linkage Difference Closed
Output Multipliers**

