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Chapter 21

A Taxonomy of Extractions*

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Abstract

Much of the work that has been done on economic linkage measures that are derived from input-output data has been or can be cast in the framework of partitioned matrices. In this paper we explore the ramifications of alternative "hypothetical extractions" within that framework. We examine all possible extractions and speculate on the plausibility of the economic stories that might underpin them, as well as whether or not they measure any interesting kind of linkage. We demonstrate that a number of alternative extractions produce identical results for certain measures of sectoral importance. In numerical illustrations, we find that an even larger number of the extractions produce identical rankings of sectors.

1. Introduction

There was a famous exchange between J.H. Clapham and A.C. Pigou in the prestigious Economic Journal in 1922 on what Clapham called "empty economic boxes" (Clapham, 1922; Pigou, 1922). Clapham complained about three industrial categories (boxes) that had been created by academic economists, namely (a) diminishing return industries, (b) constant return industries and (c) increasing return industries. It was his contention that these categories were impossible to fill with examples from the real world.[1]

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*An earlier version of this paper was presented at the Regional Science Association International meetings in Buffalo, New York, in November 1997. We thank Erik Dietzenbacher for perceptive comments on that version and more recent drafts. We have chosen to include it in this volume because of Ben Stevens's abiding interest in, curiosity about and contributions to input-output models; he never tired of calling them "putt-putt" models (as in the sound made by a small inboard-motor boat).

[1] In fact, Pigou (p. 461) characterizes Clapham's position as "... first, that his economic boxes, as long as they are empty, cannot have practical usefulness; secondly, that, even if they were filled, they would not have practical usefulness; thirdly, that they cannot be filled." He (Pigou) then proceeds to argue against these positions.

It seemed to us that it would be possible to create several categorical boxes through alternative ways of "hypothetically extracting" one or more sectors in an interindustry framework in order to assess their "linkage" with or "importance" to the economy from which they were extracted. We create those boxes and the look at what kinds, if any, of real-world examples might fit inside each of them. In spatial input-output models, it is also possible to consider "extracting" one or more regions to assess their importance to the multiregional economy.[2] We will generally concentrate on the sectoral scenario in this chapter; conversion to the regional setting is reasonably straightforward.[3]

2. Linkage measures

The information contained in input-output *accounts*, as in the direct input coefficients matrix, $\mathbf{A} = \mathbf{Z}(\hat{\mathbf{x}})^{-1}$, is a snapshot of economic interconnections among sectors in an economy at a given point in time. For an n-sector economy, $\mathbf{Z}$ is an $(n \times n)$ matrix of intra- and intersectoral flows of goods and services and $\mathbf{x}$ is an n-element vector of sectoral gross outputs. These facts about the economy may be of value in their own right, irrespective of the input-output *model* in which they are usually embedded for the purpose of conducting input-output *analysis*—via the Leontief inverse relationship $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$, where $\mathbf{y}$ is an n-element vector of final demands.

2.1 Backward Linkages

One early *direct backward linkage* measure consisted of column sums of the $\mathbf{A}$ matrix, $\mathbf{i}'\mathbf{A}$.[4] In terms of flows ($\mathbf{Z}$, not $\mathbf{A}$), this is simply the value of total intermediate inputs for sector $j\,(\sum_{i=1}^{n} z_{ij})$ as a proportion of the value of j's total output (x_j). This definition, in flow terms, was first proposed by Chenery and Watanabe (1958). To capture both direct and indirect linkages in an economy, column sums of the Leontief inverse, $\mathbf{i}'(\mathbf{I} - \mathbf{A})^{-1}$, were proposed as a *total backward* linkage measure (Rasmussen, 1957); these are commonly known as output multipliers. It is obvious, but nonetheless perhaps worth

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[2] Finally, specific sectors in specific regions could be extracted in interregional or multiregional input-output models.

[3] In a different context—the spatial hierarchical decomposition of multipliers—Sonis, Hewings, and Miyazawa (1997) have set out the basic mathematical formulae for a larger number of "boxes." The extractions that we examine below appear as "compartments" in some of their boxes. (See also Sonis and Hewings, 1999.)

[4] Throughout this chapter, we assume that the summation vector, $\mathbf{i}$, is a column vector of 1s that is always conformable for the operations in which it appears.

noting, that by embracing this measure of total backward linkage, one is buying into the entire demand-driven story behind that input-output model, $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$—specifically, that changes in final demands induce changes in outputs via production functions with fixed input coefficients (along with no bottlenecks or capacity limitations, etc.). Following many others, we call this the Leontief *quantity* model.

We note in passing that there is some disagreement in the literature on whether or not the on-diagonal elements in $\mathbf{A}$ or $(\mathbf{I} - \mathbf{A})^{-1}$ should be included or netted out of the summations (see, for example, Harrigan and McGilvray, 1988). To the extent that these "internal linkages" constitute part of Hirschman's (1958, p. 100) "input-provision, derived demand ... effects," they are appropriately included. On the other hand, if one is specifically interested in a sector's "backward dependence" on or linkage to the rest of the economy, they should be omitted. We return to this later.

Also, various normalizations of these measures have been proposed and used in empirical studies. For example $\mathbf{i}'\mathbf{A}/[(\mathbf{i}'\mathbf{A}\mathbf{i})/n] = n\mathbf{i}'\mathbf{A}/\mathbf{i}'\mathbf{A}\mathbf{i}$ produces a row vector of direct backward linkage indicators whose (simple) average value is unity—so that sectors with "above average" backward linkages have indices that are greater than one and that those with "below average" linkages have indices that are less than one. The same logic generates $n\mathbf{i}'(\mathbf{I} - \mathbf{A})^{-1}/\mathbf{i}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{i}$ as a normalized total backward linkage index.[5]

2.2 Forward Linkages

An early measure of *direct forward linkage* was also proposed, namely the row sums of the $\mathbf{A}$ matrix, $\mathbf{Ai}$, along with an associated *total forward linkage* measure, the row sums of the Leontief inverse, $(\mathbf{I} - \mathbf{A})^{-1}\mathbf{i}$.[6] Both of these have been viewed with skepticism, because they quantify responses to peculiar stimuli—a simultaneous increase of one unit in the gross outputs of every sector in the case of $\mathbf{Ai}$ and an increase of one unit in the final demands of every sector in the case of $(\mathbf{I} - \mathbf{A})^{-1}\mathbf{i}$.[7]

[5] This is Rasmussen's "Index of the Power of Dispersion."

[6] In the normalized form, $n(\mathbf{I} - \mathbf{A})^{-1}\mathbf{i}/\mathbf{i}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{i}$, this is Rasmussen's "Index of Sensitivity of Dispersion."

[7] Among the first to make an issue of weightings in linkage measures was Laumas (1976). Others before him (e.g., Hazari, 1970; Diamond, 1974), however, had used sets of weights other than unit vectors. There is some question about the appropriateness of any normalization by the average unit-weighted multiplier, $\mathbf{i}'(\mathbf{I} - \mathbf{A})^{-1}\mathbf{i}/n$ as in Rasmussen's Index of Sensitivity of Dispersion or his Index of the Power of Dispersion. This is because this "average" multiplier is not a true average multiplier for the economy (which would have to be derived using weights denoting sector size) and, therefore, has no convincing economic or statistical interpretation.

This dissatisfaction led to the suggestion that elements from the Ghosh model (often called the supply-driven or supply-side model in this literature) would be more appropriate as forward linkage measures (Beyers, 1976; Jones, 1976). The heart of this model is a direct output coefficients matrix, $\mathbf{B} = (\hat{\mathbf{x}})^{-1}\mathbf{Z}$, in which b_{ij} indicates the proportion of sector *i*'s output that goes to sector *j*; the row sums of this matrix, **Bi**, were proposed as measures of *direct forward linkage*. In terms of flows (**Z**, not **B**), this is simply the value of total intermediate sales for sector i ($\sum_{j=1}^{n} z_{ij}$) as a proportion of the value of *i*'s total output (x_i).[8] In addition, row sums of the Ghosh inverse, namely $(\mathbf{I} - \mathbf{B})^{-1}\mathbf{i}$, were suggested to measure *total forward linkages*. As with direct backward linkages, using **B** for direct forward linkage measures does not require any acceptance of the *model* that underpins the Ghosh inverse, namely $\mathbf{x}' = \mathbf{w}'(\mathbf{I} - \mathbf{B})^{-1}$, where $\mathbf{w}'$ is a row vector of value added amounts. In this model, as Dietzenbacher (1997) makes clear, output *values* change linearly with changes in *prices* of primary inputs. He suggests abandoning the label "supply-driven input-output model" and using, instead, "Ghosh *price* model." We follow this lead.[9]

Measures of direct linkage are simply transformations of input-output account data, but they are not always sufficiently interesting because they do not capture much of the inherent complexity of an economy. By contrast, measures of total linkage are by definition designed to capture direct plus indirect effects,[10] but they rest on a structural *models* that underpins them. Arguments have been made suggesting that one can use the row sums in **B** and $(\mathbf{I} - \mathbf{B})^{-1}$ without necessarily embracing the whole Ghosh model causal structure.[11] For example, **A** and **B** are similar matrices,

$$\mathbf{B} = (\hat{\mathbf{x}})^{-1}\mathbf{A}(\hat{\mathbf{x}}) \tag{1}$$

and so the Leontief and Ghosh inverses are also similar;

$$(\mathbf{I} - \mathbf{B})^{-1} = (\hat{\mathbf{x}})^{-1}(\mathbf{I} - \mathbf{A})^{-1}(\hat{\mathbf{x}}) \tag{2}$$

Thus, direct forward linkages, **Bi**, could be calculated as $(\hat{\mathbf{x}})^{-1}\mathbf{A}\hat{\mathbf{x}}\mathbf{i} = (\hat{\mathbf{x}})^{-1}\mathbf{A}\mathbf{x}$ and total forward linkages, $(\mathbf{I} - \mathbf{B})^{-1}\mathbf{i}$, could be found as $(\hat{\mathbf{x}})^{-1}(\mathbf{I} - \mathbf{A})^{-1}\hat{\mathbf{x}}\mathbf{i} = (\hat{\mathbf{x}})^{-1}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}$, thereby completely avoiding the Ghosh

[8] Again, this was first proposed in Chenery and Watanabe (1958). They did not work with the B matrix. It first appeared in Ghosh's February 1958 article. Chenery and Watanabe note that an earlier version of their article was presented to the Econometric Society in December 1956, and, considering publication lags, there is no reason to believe that they knew of Ghosh's work.

[9] As with backward linkage measures, inclusion or exclusion of on-diagonal elements is an issue, and normalizations are usual.

[10] In models that are closed with respect to households, *induced* effects are also included.

[11] See Oosterhaven (1988, 1996), Dietzenbacher, van der Linden, and Steenge (1993), Dietzenbacher (1997), and de Mesnard (1997), for example. A more skeptical view is presented in Cella (1988).

price model and the economic assumptions driving it. This is little comfort, however, since the required weights—x_j/x_i—across row i of $\mathbf{A}$ or $(\mathbf{I}-\mathbf{A})^{-1}$ do not seem to have an appealing economic interpretation. Nonetheless, in Section 5 we discuss the relative merits of the Ghosh model in greater depth.

3. The partitioning structure

Consider the standard representation of an n-sector input-output technical coefficients matrix that has been partitioned so that k sectors ($k < n$) are shown in the upper left square submatrix, identified as $\mathbf{A}_{11}$. That is,

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right] \tag{3}$$

Then the Leontief inverse of this partitioned matrix can be expressed as[12]

$$\mathbf{L} = (\mathbf{I}-\mathbf{A})^{-1} = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}(\mathbf{I}+\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22}) \end{array}\right] \tag{4}$$

where $\mathbf{H} = (\mathbf{I}-\mathbf{A}_{11}-\mathbf{A}_{12}\boldsymbol{\alpha}_{22}\mathbf{A}_{21})^{-1}$ and $\boldsymbol{\alpha}_{22} = (\mathbf{I}-\mathbf{A}_{22})^{-1}$. Final demands and gross outputs can be partitioned similarly, so that

$$\mathbf{y} = \left[\begin{array}{c} \mathbf{y}_1 \\ \hline \mathbf{y}_2 \end{array}\right] \quad \text{and} \quad \mathbf{x} = \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{x}_2 \end{array}\right]$$

and

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{x}_2 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}(\mathbf{I}+\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22}) \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hline \mathbf{y}_2 \end{array}\right] \tag{5}$$

[12] We part here from the notation of some others in the field, in order to avoid potential confusion. For example, we have opted for $\boldsymbol{\alpha}_{11}$ instead of $\mathbf{B}_{11}$ to denote the Leontief inverse of $\mathbf{A}_{11}$ because we have reserved $\mathbf{B}$ for the Ghosh allocation matrix. We also do not use $\mathbf{L}_{11}$ since that would identify the upper left partition of the Leontief inverse, and in general, $\mathbf{L}_{11} \neq (\mathbf{I}-\mathbf{A}_{11})^{-1}$ as shown in (4). Similarly, $\boldsymbol{\alpha}_{22} = (\mathbf{I}-\mathbf{A}_{22})^{-1}$. Alternative expressions for the partitioned inverse are possible; see footnote 31 in Appendix A.

This representation provides a useful framework in which to examine various kinds of possible ''hypothetical extraction'' linkage measures.[13] The original idea (Paelinck, de Caevel, and Degueldre, 1965; Strassert, 1968) was to try to quantify how much an economy's total output would decrease if a particular sector, say the jth, were not present. In input-output terms for an n-sector economy, this was modeled by deleting row and column j from the $\mathbf{A}$ matrix. Using $\bar{\mathbf{A}}$ for the $[(n-1) \times (n-1)]$ matrix without sector j and $\bar{\mathbf{y}}$ for the correspondingly reduced final demand vector, output in the ''reduced'' economy is found as $\bar{\mathbf{x}} = (\bar{\mathbf{I}} - \bar{\mathbf{A}})^{-1}\bar{\mathbf{y}}$. In the full n-sector model, output is $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$; so $\mathbf{i}'\mathbf{x} - i'\bar{\mathbf{x}}$ is one aggregate measure of the economy's loss if sector j disappears.

Assume that the one or more sectors (or regions) to be extracted from the economic system are those that occupy the first k rows and columns. For concreteness in what follows, and to allow us to examine comparable ''stories,'' we will generally assume that we are speaking of sectors (not regions) and that only one sector is being extracted (i.e., $k = 1$). This is consistent with much of the ''key sector'' literature, in which a measure of the relative importance of any particular sector in an economy is found by extracting that sector.

There are essentially two issues in the literature on this kind of linkage measurement. First, there is the objective of providing a comprehensive kind of *total linkage* (economic ''importance'') indicator for a sector—$\mathbf{i}'\mathbf{x} - i'\bar{\mathbf{x}}$ (or variants) is one such measure.[14] Secondly, researchers have explored the question of how a total linkage measure might be disaggregated into (or built up from) *backward*

[13] Cella (1984) appears to be the first to have used this kind of partitioned matrix structure in the context of the interindustry linkage measurement. Using the same structuring, Miller (1966) earlier measured interregional feedback effects. Similar partitionings of the input-output structure have appeared in several places, including Ghosh (1960), Moses (1960), Stone (1961), Yamada (1961), Miller (1963), and Rutsch (1964). Additionally, Leontief (1967) used a partitioned model to describe an aggregation procedure for input-output models, and essentially used the same algebra that also appeared later in Pyatt (1985, 1989). The hypothetical extraction approach for sectors was first published in English in Schultz (1976, 1977).

[14] Hirschman (1958, Chap. 6) originally suggested the idea of measuring the ''total linkage'' of a sector. He cites two major works on linkage measurement, by Chenery and Watanabe and by Rasmussen. His reference to Chenery and Watanabe is to their 1956 Econometric Society paper (see footnote 8, above) ''. . . to be published in *Econometrica*.'' As for Rasmussen, Hirschman cites a version of *Studies in Inter-Sectoral Relations* published by Einar Harcks in Copenhagen in 1956. This must be a precursor to the 1957 North-Holland edition (also under the Einar Harcks imprint), identified as a ''second printing'' and which says, at the end of the Acknowledgements, ''The manuscript was completed towards the end of 1954.'' Many authors subsequently attempted to discover appropriate methods for measuring total linkages. See Hewings (1982) or Harrigan and McGilvray (1988) for reviews of much of the material on this topic through the 1970s and mid-1980s, respectively.

and *forward linkage* components. The partitioned form of the Leontief inverse in (4) and (5) suggests some fairly straightforward parallels to the early descriptions of backward and forward linkages as column and row sums from a Leontief inverse. Meller and Marfán (1981) were the first to measure forward linkages as a function of Hirschman's "pressure of demand." They did so using a "top-down" approach—identifying total linkages through a kind of extraction procedure, backward linkages as column sums from the Leontief inverse (possibly weighted) and forward linkages as a residual (the difference between total and backward linkage). Cella (1984) made this identification easier by formalizing the partitioned matrix approach.[15]

From a purely mathematical viewpoint, there are three kinds of "extraction" cases to be investigated. One can remove from (replace with null matrices) the partitioned $\mathbf{A}$ matrix in (3):

(1) all three submatrices in which sector 1 plays a role;

(2) pairs of the three submatrices; and

(3) only one of them.

We examine these options in turn.

4. Leontief linkages

Hirschman (1958) is generally regarded as providing the original impetus for linkage studies. In that book, he suggested that forward linkage "must always be accompanied by backward linkage, which is a result of the 'pressure of demand.' . . . [Forward linkage] acts as an important and powerful reinforcement to backward linkage" (pp. 116–117). Due to Hirschman's focus on demand pressures, most early linkage analysts focused on attributes of the direct requirements matrix and the Leontief inverse. In classifying the various types of extraction approaches, we begin from this perspective as well. As noted above, we assume for simplicity that only one industry is extracted ($k = 1$) so that, for example, $\mathbf{x}_1$ and $\mathbf{y}_1$ are scalars.

[15] Others (e.g., Harrigan and McGilvray, 1988, and Clements, 1990) subsequently pointed out different ways to decompose total linkages from a Leontief quantity model (see Section 4). By contrast, "bottom-up" approaches have also been proposed. Hübler (1979) suggests that column sums from a "combined" model, namely $[\mathbf{I} - (\mathbf{0.5})(\mathbf{A} + \mathbf{B}')]^{-1}$, constitute total linkage measures. Loviscek's (1982) article is a nightmare of typos and confusing notation, but the kindest interpretation (noted in Oosterhaven, 1988) is that he proposes using column sums from $(\mathbf{0.5})[(\mathbf{I} - \mathbf{A})^{-1} + (\mathbf{I} - \mathbf{B})^{-1}]$ to measure total linkages. Neither of these combinations has a convincing theoretical underpinning.

Case 1. Extract all three matrices in which sector 1 has an influence.

Set $\mathbf{A}_{11} = \mathbf{A}_{12} = \mathbf{A}_{21} = \mathbf{0}$, so

$$\mathbf{A}^1 = \left[\begin{array}{c:c} \mathbf{0} & \mathbf{0} \\ \hdashline \mathbf{0} & \mathbf{A}_{22} \end{array}\right] \tag{6}$$

Then, the Leontief inverse appears as

$$\mathbf{L}^1 = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \\ \hdashline \mathbf{0} & \boldsymbol{\alpha}_{22} \end{array}\right] \tag{7}$$

This is the method of extraction originally conceived by Paelinck, de Caevel, and Degueldre (1965), and later employed by Strassert (1968), Schultz (1976, 1977), Meller and Marfán (1981), Milana (1985), and Heimler (1991).[16] The pre-extraction total output vector is given in (5). Using appropriately numbered superscripts to denote the extraction case under consideration, we have from (7)

$$\mathbf{x}^1 = \left[\begin{array}{c} \mathbf{x}_1^1 \\ \hdashline \mathbf{x}_2^1 \end{array}\right] = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \\ \hdashline \mathbf{0} & \boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hdashline \mathbf{y}_2 \end{array}\right] \tag{8}$$

Then from (5) and (8),

$$\Delta\mathbf{x}^1 = \left[\begin{array}{c} \mathbf{x}_1 - \mathbf{x}_1^1 \\ \hdashline \mathbf{x}_2 - \mathbf{x}_2^1 \end{array}\right] = \left[\begin{array}{c} \Delta\mathbf{x}_1^1 \\ \hdashline \Delta\mathbf{x}_2^1 \end{array}\right] = \left[\begin{array}{c:c} \mathbf{H} - \mathbf{I} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hdashline \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hdashline \mathbf{y}_2 \end{array}\right]$$

$$= \left[\begin{array}{c:c} \boldsymbol{\Delta}_{11}^{1L} & \boldsymbol{\Delta}_{12}^{1L} \\ \hdashline \boldsymbol{\Delta}_{21}^{1L} & \boldsymbol{\Delta}_{22}^{1L} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hdashline \mathbf{y}_2 \end{array}\right] \tag{9}$$

where "**L**" denotes extractions from the Leontief model. (In Section 5 we will use "**G**" for extractions of the Ghosh model.)

This is one comprehensive measure of sector 1's importance to the economy; it reflects removal of all connections—forward, backward, and internal. Since sector 1 ceases to exist ($\mathbf{A}_{11} = \mathbf{A}_{12} = \mathbf{A}_{21} = \mathbf{0}$), then $\mathbf{x}_1^1 = \mathbf{0}$ and the amount of its output that goes to satisfy final demand for sector 1 goods is also zero; then the (original) amount of $\mathbf{y}_1$ would have to be satisfied by imports. In this

[16] Groenewold, Hagger, and Madden (1993) call this scenario "shut-down of [the] industry."

scenario, then, it would be appropriate to measure the importance of sector 1 in the total economy from which it is "completely extracted" by $\mathbf{i}'\Delta\mathbf{x}^{\mathbf{1}} + \mathbf{y}_1 = \mathbf{i}'\mathbf{x} - \mathbf{i}'\mathbf{x}_2^1 = \mathbf{x}_1 + \mathbf{i}'\Delta\mathbf{x}_2^1$. To examine the importance of the excluded sector to just those sectors that remain, it is the vector $\Delta\mathbf{x}_2^1 = \mathbf{x}_2 - \mathbf{x}_2^1$ that is of interest, so the appropriate measure is $\mathbf{i}'\Delta\mathbf{x}_2^1 = \mathbf{i}'\Delta_{21}^{1L}\mathbf{y}_1 + \mathbf{i}'\Delta_{22}^{1L}\mathbf{y}_2$.[17] This is used, for example, in Schultz (1977).

Case 2. Extract two of the three matrices in which sector 1 has any influence.

Clearly, there are three ways of doing this.

Case 2a. $\mathbf{A}_{12} = \mathbf{A}_{21} = \mathbf{0}$. Here

$$\mathbf{A}^{2a} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right] \tag{10}$$

and

$$\mathbf{L}^{2a} = \left[\begin{array}{c|c} \alpha_{11} & \mathbf{0} \\ \hline \mathbf{0} & \alpha_{22} \end{array}\right] \tag{11}$$

The difference between gross outputs in the economy without and with sector 1 extracted in this manner is

$$\Delta\mathbf{x}^{2a} = \left[\begin{array}{c} \Delta\mathbf{x}_1^{2a} \\ \hline \Delta\mathbf{x}_2^{2a} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H} - \alpha_{11} & \mathbf{H}\mathbf{A}_{12}\alpha_{22} \\ \hline \alpha_{22}\mathbf{A}_{21}\mathbf{H} & \alpha_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\alpha_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hline \mathbf{y}_2 \end{array}\right] \tag{12}$$

In this case, all of sector 1's linkages to the rest of the economy are eliminated; it differs from Case 1 only by the retention of $\mathbf{A}_{11}$ (intrasectoral linkage for sector 1). The sum of the elements in $\Delta\mathbf{x}^{2a}$, $\mathbf{i}'(\mathbf{x}^{2a})$, is the total linkage measure presented by Cella (1984). He argues that this particular "extraction," setting $\mathbf{A}_{12} = \mathbf{0}$ and $\mathbf{A}_{21} = \mathbf{0}$, sets up the appropriate measure of "the quantities of *n* goods directly and indirectly stimulated by the intermediate functions (both as purchaser and as supplier)" of sector 1 (p. 74). Miller (1966, 1969), Miller and Blair (1983), and Dietzenbacher, van der Linden, and Steenge (1993)—among

[17] Throughout this chapter we apply the vector of sectoral final demands as the set of weights that are used to derive measures of Leontief linkage and, in Section 5, sectoral value added for Ghosh linkages. The authors we cite often did not. Meller and Marfán (1981), for example, actually used physical labor proportions as weights for combining the effects of the sectors. Although not original (cf., Hazari, 1970; Diamond, 1974; McGilvray, 1977; Rao and Harmston, 1979), their application of physical labor proportions is an excellent demonstration of how one can use readily available data as an alternative to the sometimes difficult-to-derive final demand figures.

others—applied this structure in a spatial (interregional) setting to measure interregional feedback effects (interregional linkages).[18]

Cella developed this approach partly in response to Schultz and to Meller and Marfán, because he believed that they had accounted for too little and too much linkage, respectively, in using (8).[19] He suggested this modification because it removes the extracted sector's internal linkage to itself [in the upper left submatrix in (12)], and one might argue that an industry's self-supply can be considered both a forward and backward link.[20]

Furthermore, Cella proposed a decomposition of this total linkage indicator into forward and backward linkage components. Specifically, he suggested that the two submatrices in the left half of the partitioned inverse serve to capture backward linkages, namely

$$BL_1 = \mathbf{i}'(\mathbf{H} - \boldsymbol{\alpha}_{11})\mathbf{y}_1 + \mathbf{i}'(\boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H})\mathbf{y}_1$$

and that forward linkages are measured in the two submatrices in the right half of that inverse, as

$$FL_1 = \mathbf{i}'(\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22})\mathbf{y}_2 + \mathbf{i}'(\boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22})\mathbf{y}_2$$

This reflects the logical conditions that sector 1's backward linkage is zero if and only if $\mathbf{A}_{21} = \mathbf{0}$ (making $\mathbf{H} = \boldsymbol{\alpha}_{11}$) and its forward linkage is zero if and only if $\mathbf{A}_{12} = \mathbf{0}$.

These definitions of forward and backward linkage components from the partitioned structure in (12) have drawn criticism. For example, Clements (1990) argues that $\mathbf{i}'(\boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22})\mathbf{y}_2$ belongs as a third term in BL_1, leaving only $\mathbf{i}'(\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22})\mathbf{y}_2$ as FL_1. A more fundamental disagreement appears initially to have been raised by Guccione (1986), namely that the two terms in Cella's FL_1 are in fact more appropriately viewed as the *backward* linkage of sector(s) 2—the rest of the economy—on 1 (see also Cella, 1986, for some reactions to this and other criticisms). Dietzenbacher, van der Linden and Steenge (1993) have reiterated this point of view, insisting that only backward linkages are to be found from the Leontief model and that (harking back to Beyers, 1976, and Jones,

[18] Appendix A contains a note on interregional feedbacks, linkages, and normalizations.

[19] Cella (1984, p. 79) suggests that he is "sharpening up" the approach of Schultz.

[20] The magnitude of this internal linkage effect depends in part on the level of aggregation in the input-output model. If sector 1 is "manufacturing," this effect will be large; if sector 1 is "brass bolts," it is likely to be very small.

1976) forward linkage measures must come from elements of the Ghosh price model.[21]

Case 2b. $\mathbf{A_{11}} = \mathbf{A_{21}} = \mathbf{0}$. Here

$$\mathbf{A}^{2b} = \left[\begin{array}{c:c} \mathbf{0} & \mathbf{A_{12}} \\ \hdashline \mathbf{0} & \mathbf{A_{22}} \end{array}\right] \tag{13}$$

and

$$\mathbf{L}^{2b} = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{A_{12}}\boldsymbol{\alpha}_{22} \\ \hdashline \mathbf{0} & \boldsymbol{\alpha}_{22} \end{array}\right] \tag{14}$$

The difference between gross outputs in the economy without and with sector 1 extracted in this manner is

$$\Delta\mathbf{x}^{2b} = \left[\begin{array}{c} \Delta\mathbf{x}_1^{2b} \\ \hdashline \Delta\mathbf{x}_2^{2b} \end{array}\right] = \left[\begin{array}{c:c} \mathbf{H} - \mathbf{I} & (\mathbf{H} - \mathbf{I})\mathbf{A_{12}}\boldsymbol{\alpha}_{22} \\ \hdashline \boldsymbol{\alpha}_{22}\mathbf{A_{21}}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A_{21}}\mathbf{H}\mathbf{A_{12}}\boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hdashline \mathbf{y}_2 \end{array}\right] \tag{15}$$

This can be viewed as one measure of the strength of sector 1's backward linkage, since all intermediate inputs into the sector are removed. This measure was also presented in Szyrmer and Walker (1983). They suggest that the *i*th element in $\Delta\mathbf{x}^{\mathbf{2b}}$ is a "measure of the total output from sector *i* created by the input linkage between *i* and the extracted sector [which is sector 1 in (15)]. As such, it is the total flow from *i* to the extracted sector, not just that part associated with the demand for [the extracted sector's goods]" (Szyrmer and Walker, 1983, p. 14; see also Szyrmer, 1986, 1992). The structure is also used by Dietzenbacher and van der Linden (1997) to generate their preferred (spatial) backward linkage measure.

Groenewold, Hagger, and Madden (1993, p. 177) use this variant to model the "elimination of domestic raw-material purchases" by a sector, while

[21] Cella (1984) seems to have been the first to argue that indices from Leontief and Ghosh models cannot be combined, basically because of inconsistent stability assumptions about the coefficient matrices that underpin the two models. (See also Cella, 1988.) This came to be known as the "joint stability" problem. (Dietzenbacher, 1997, identifies some of the most prominent literature in this debate.) Essentially, the issue is this: Using fixed, base-year data, $\mathbf{A}(\mathbf{0}) = \mathbf{Z}(\mathbf{0})[\hat{\mathbf{x}}(\mathbf{0})]^{-1}$, in a Leontief quantity model to assess, *ex ante*, the impact of a "new" $\mathbf{y}(\mathbf{1})$ gives $\mathbf{x}(\mathbf{1}) = [\mathbf{I} - \mathbf{A}(\mathbf{0})]^{-1}\mathbf{y}(\mathbf{1})$. This implies a new flow matrix, $\mathbf{Z}(\mathbf{1}) = \mathbf{A}(\mathbf{0})\hat{\mathbf{x}}(\mathbf{1})$, and this, in turn, implies a new output coefficients matrix, $\mathbf{B}(\mathbf{1}) = [\hat{\mathbf{x}}(\mathbf{1})]^{-1}\mathbf{Z}(\mathbf{1})$; in general, $\mathbf{B}(\mathbf{1}) \neq \mathbf{B}(\mathbf{0}) = [\hat{\mathbf{x}}(\mathbf{0})]^{-1}\mathbf{Z}(\mathbf{0})$. So the assumption of a stable $\mathbf{A}$ matrix is not compatible with a stable $\mathbf{B}$ matrix, and vice-versa. Nonetheless, evidence is mounting that shows output coefficients are not much more unstable than input coefficients (de Mesnard, 1997).

maintaining the output distribution patterns over the remaining sectors. In a spatial model, it would represent the effect of eliminating all shipments into a region (region 1), as well the intraregional supplies of that region.

Case 2c. $\mathbf{A_{11}} = \mathbf{A_{12}} = \mathbf{0}$. Here

$$\mathbf{A}^{2c} = \left[\begin{array}{c:c} \mathbf{0} & \mathbf{0} \\ \hdashline \mathbf{A_{21}} & \mathbf{A_{22}} \end{array}\right] \tag{16}$$

which has the Leontief inverse

$$\mathbf{L}^{2c} = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \\ \hdashline \boldsymbol{\alpha}_{22}\mathbf{A_{21}} & \boldsymbol{\alpha}_{22} \end{array}\right] \tag{17}$$

and yields the following total change in output:

$$\Delta\mathbf{x}^{2c} = \left[\begin{array}{c} \Delta\mathbf{x}_1^{2c} \\ \hdashline \Delta\mathbf{x}_2^{2c} \end{array}\right] = \left[\begin{array}{c:c} \mathbf{H} - \mathbf{I} & \mathbf{H}\mathbf{A_{12}}\boldsymbol{\alpha}_{22} \\ \hdashline \boldsymbol{\alpha}_{22}\mathbf{A_{21}}(\mathbf{H} - \mathbf{I}) & \boldsymbol{\alpha}_{22}\mathbf{A_{21}}\mathbf{H}\mathbf{A_{12}}\boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y_1} \\ \hdashline \mathbf{y_2} \end{array}\right] \tag{18}$$

Parallel to Case 2b, this can be viewed as one measure of the strength of sector 1's forward linkage, since all intermediate shipments from the sector are removed. Groenewold, Hagger, and Madden (1987) discuss this measure as a partial improvement over that given in Case 1, which they criticize for ''over-counting'' the individual effects associated with a sector's extraction. They call this ''relocation of industry'' (also in Groenewold, Hagger, and Madden, 1993), equivalent to a sector's moving to outside of the original spatial system but continuing to purchase intermediate inputs from the remaining sectors (in the same proportions as prior to relocation, as reflected in $\mathbf{A_{21}} \neq \mathbf{0}$). Alternatively, one could view this as modeling a scenario in which all local sales are replaced by exports.

Case 3. Extract only one submatrix in which sector 1 has an influence.

Again, there are three possibilities.

Case 3a. $\mathbf{A_{12}} = \mathbf{0}$, so

$$\mathbf{A}^{3a} = \left[\begin{array}{c:c} \mathbf{A_{11}} & \mathbf{0} \\ \hdashline \mathbf{A_{21}} & \mathbf{A_{22}} \end{array}\right] \tag{19}$$

and

$$\mathbf{L}^{3a} = \left[\begin{array}{c:c} \alpha_{11} & \mathbf{0} \\ \hdashline \alpha_{22}\mathbf{A}_{21}\alpha_{11} & \alpha_{22} \end{array}\right] \tag{20}$$

This yields the following total change in output:

$$\Delta\mathbf{x}^{3a} = \left[\begin{array}{c} \Delta\mathbf{x}_1^{3a} \\ \hdashline \Delta\mathbf{x}_2^{3a} \end{array}\right] = \left[\begin{array}{c:c} \mathbf{H}-\alpha_{11} & \mathbf{H}\mathbf{A}_{12}\alpha_{22} \\ \hdashline \alpha_{22}\mathbf{A}_{21}(\mathbf{H}-\alpha_{11}) & \alpha_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\alpha_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hdashline \mathbf{y}_2 \end{array}\right] \tag{21}$$

This too can be viewed as a plausible measure of sector 1's forward linkage. It differs from Case 2c only in that now intrasectoral sales and purchases are retained. Indeed, it may be a more appealing measure than that in Case 2c, since industries have a propensity to sell intrasectorally and are rather unlikely to eliminate those sales. A possible economic scenario that would generate this structure is when a sector makes a decision to sell only for export, except for deliveries to itself. An example might be the use of coking coal as fuel by the coking coal mining industry in southern West Virginia, whose product is otherwise diverted to international sales in response to increased foreign demand.

Case 3b. $\mathbf{A}_{21} = \mathbf{0}$, or

$$\mathbf{A}^{3b} = \left[\begin{array}{c:c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hdashline \mathbf{0} & \mathbf{A}_{22} \end{array}\right] \tag{22}$$

with the associated Leontief inverse

$$\mathbf{L}^{3b} = \left[\begin{array}{c:c} \alpha_{11} & \alpha_{11}\mathbf{A}_{12}\alpha_{22} \\ \hdashline \mathbf{0} & \alpha_{22} \end{array}\right] \tag{23}$$

The total change in output is

$$\Delta\mathbf{x}^{3b} = \left[\begin{array}{c} \Delta\mathbf{x}_1^{3b} \\ \hdashline \Delta\mathbf{x}_2^{3b} \end{array}\right] = \left[\begin{array}{c:c} \mathbf{H}-\alpha_{11} & (\mathbf{H}-\alpha_{11})\mathbf{A}_{12}\alpha_{22} \\ \hdashline \alpha_{22}\mathbf{A}_{21}\mathbf{H} & \alpha_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\alpha_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y}_1 \\ \hdashline \mathbf{y}_2 \end{array}\right] \tag{24}$$

This measure is another indicator of sector 1's backward linkage. It differs from Case 2b only in retaining intrasectoral transactions (as in Case 3a, above, also). Note that Case 3b for sector 1 is the same as Case 3a would be for sector 2; similarly, Case 3a for sector 1 is the same as Case 3b for sector 2. Here, as in Case 2b, an economic scenario might be that the sector finds itself forced to purchase imports only, substituting completely for locally produced inputs.

Case 3c. $\mathbf{A_{11}} = \mathbf{0}$, so

$$\mathbf{A}^{3c} = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{A_{12}} \\ \hline \mathbf{A_{21}} & \mathbf{A_{22}} \end{array}\right] \tag{25}$$

which has the Leontief inverse

$$\mathbf{L}^{3c} = \left[\begin{array}{c|c} \mathbf{\Phi} & \mathbf{\Phi A_{12}}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A_{21}\Phi} & \boldsymbol{\alpha}_{22}(\mathbf{I} + \mathbf{A_{21}\Phi A_{12}}\boldsymbol{\alpha}_{22}) \end{array}\right] \tag{26}$$

where $\mathbf{\Phi} = (\mathbf{I} - \mathbf{A_{12}}\boldsymbol{\alpha}_{22}\mathbf{A_{21}})^{-1}$. The change in output is

$$\Delta\mathbf{x}^{3c} = \left[\begin{array}{c} \Delta\mathbf{x}_1^{3c} \\ \hline \Delta\mathbf{x}_2^{3c} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{H} - \mathbf{\Phi} & (\mathbf{H} - \mathbf{\Phi})\mathbf{A_{12}}\boldsymbol{\alpha}_{22} \\ \hline \boldsymbol{\alpha}_{22}\mathbf{A_{21}}(\mathbf{H} - \mathbf{\Phi}) & \boldsymbol{\alpha}_{22}\mathbf{A_{21}}(\mathbf{H} - \mathbf{\Phi})\mathbf{A_{12}}\boldsymbol{\alpha}_{22} \end{array}\right] \left[\begin{array}{c} \mathbf{y_1} \\ \hline \mathbf{y_2} \end{array}\right] \tag{27}$$

This does not seem to measure a particularly interesting kind of linkage—backward, forward, or total—between the extracted sector and the rest of the economy. Rather, it is a measure of that sector's internal linkage. Moreover, it is not easy to imagine a reasonable underlying economic scenario for this case in which only intrasectoral or intraregional shipments are eliminated. One possibility might be if the price of an industry's good rises to the point that it becomes unprofitable for the sector to consume its own production; for example, if the metallurgical coal sector finds it more cost effective to buy oil for its fuel stock needs rather than consume the coal it produces. That economic scenario would be modeled by this ''extraction,'' but it is not a very meaningful linkage measure.

5. Ghosh linkages and some comparisons

There are the same seven possibilities for extractions from a partitioned version of the Ghosh price model. It has been argued (Dietzenbacher, van der Linden, and Steenge, 1993; Dietzenbacher and van der Linden, 1997) that some of these provide alternative and superior measures of total forward linkage. Here, for the partitioned case, $\hat{\mathbf{x}} = \left[\begin{array}{c|c} \hat{\mathbf{x}}_1 & \mathbf{0} \\ \hline \mathbf{0} & \hat{\mathbf{x}}_2 \end{array}\right]$ and so $(\hat{\mathbf{x}})^{-1} = \left[\begin{array}{c|c} (\hat{\mathbf{x}}_1)^{-1} & \mathbf{0} \\ \hline \mathbf{0} & (\hat{\mathbf{x}}_2)^{-1} \end{array}\right]$. From the similarity of **A** and **B** (Section 2), we have

$$\mathbf{B} = \left[\begin{array}{c:c} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \hdashline \mathbf{B}_{21} & \mathbf{B}_{22} \end{array}\right] = (\hat{\mathbf{x}})^{-1}\mathbf{A}(\hat{\mathbf{x}}) = \left[\begin{array}{c:c} (\hat{\mathbf{x}}_1)^{-1}\mathbf{A}_{11}(\hat{\mathbf{x}}_1) & (\hat{\mathbf{x}}_1)^{-1}\mathbf{A}_{12}(\hat{\mathbf{x}}_2) \\ \hdashline (\hat{\mathbf{x}}_2)^{-1}\mathbf{A}_{21}(\hat{\mathbf{x}}_1) & (\hat{\mathbf{x}}_2)^{-1}\mathbf{A}_{22}(\hat{\mathbf{x}}_2) \end{array}\right] \tag{28}$$

The associated partitioned inverse is

$$\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1} = (\hat{\mathbf{x}})^{-1}(\mathbf{I} - \mathbf{A})^{-1}(\hat{\mathbf{x}}) = \left[\begin{array}{c:c} \mathbf{K} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \hdashline \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}(\mathbf{I} + \mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22}) \end{array}\right] \tag{29}$$

where

$$\mathbf{K} = (\mathbf{I} - \mathbf{B}_{11} - \mathbf{B}_{12}\boldsymbol{\beta}_{22}\mathbf{B}_{21})^{-1} = (\hat{\mathbf{x}}_1)^{-1}\mathbf{H}(\hat{\mathbf{x}}_1)$$

and

$$\boldsymbol{\beta}_{22} = (\mathbf{I} - \mathbf{B}_{22})^{-1} = (\hat{\mathbf{x}}_2)^{-1}\boldsymbol{\alpha}_{22}(\hat{\mathbf{x}}_2).$$ [22]

Value added (a row vector) can also be partitioned,

$$\mathbf{w}' = [\, \mathbf{w}_1' \;\vdots\; \mathbf{w}_2' \,]$$

so that

$$\mathbf{x}' = [\, \mathbf{x}_1' \;\vdots\; \mathbf{x}_2' \,] = [\, \mathbf{w}_1' \;\vdots\; \mathbf{w}_2' \,]\left[\begin{array}{c:c} \mathbf{K} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \hdashline \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}(\mathbf{I} + \mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22}) \end{array}\right]$$

We examine the Case 1 extraction only; others will have parallels to the Leontief model cases. Here

$$\mathbf{B}^1 = \left[\begin{array}{c:c} \mathbf{0} & \mathbf{0} \\ \hdashline \mathbf{0} & \mathbf{B}_{22} \end{array}\right] \tag{31}$$

and the Ghosh inverse is

$$\mathbf{G}^1 = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \\ \hdashline \mathbf{0} & \boldsymbol{\beta}_{22} \end{array}\right] \tag{32}$$

[22] Again, alternative expressions are also possible; see footnote 31 in Appendix A.

so $\mathbf{\Delta x}^1$ generated by this Ghosh model extraction is

$$\mathbf{\Delta x}^1 = [\,\mathbf{x}_1^1 \;\vdots\; \mathbf{x}_2^1\,] = [\,\mathbf{w}_1' \;\vdots\; \mathbf{w}_2'\,] \left[\begin{array}{c:c} \mathbf{K} - \mathbf{I} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \hdashline \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{array} \right]$$

$$= [\,\mathbf{w}_1' \;\vdots\; \mathbf{w}_2'\,] \left[\begin{array}{c:c} \mathbf{\Delta}_{11}^{1G} & \mathbf{\Delta}_{12}^{1G} \\ \hdashline \mathbf{\Delta}_{21}^{1G} & \mathbf{\Delta}_{22}^{1G} \end{array} \right] \tag{33}$$

and the parallel to the results in (9) is clear.

It is not obvious precisely what kind of sector 1 linkage is measured by the result in (33). In view of the parallel to the Leontief Case 1 in (9), $\mathbf{\Delta x}^1$ in (33) could be viewed as an alternative (but, in our view, unnecessary) total linkage measure. Dietzenbacher's (1997) interesting and generally convincing interpretation of the Ghosh model characterizes it as one in which sectoral output *values* change due to *price* changes in primary inputs (a cost-push model in which all quantities remain fixed). Given the usual application of total linkages for determining the relative stimulative importance of a sector to an economy, it seems wrong-headed to apply the Ghosh price model in this context since it fails to measure any real change in economic activity. By contrast, in the Leontief model sectoral output *quantities* change in response to changes in final demand *quantities* (a demand-pull model in which all prices remain fixed). Given this combination of viewpoints, one would analyze both a sector's stimulative importance and the economy's sensitivity to primary-input prices if one measured total linkages with both the Leontief and Ghosh models and then assessed the sector on the basis of its scores on both measures.

But one can argue that the distribution of the value change in the economy due to a sector's extraction in the economy as depicted in the difference between two Ghosh inverses remains a valid measure of the sector's forward linkage effect. Indeed, if one follows the thinking of Guccione and Gillen (1986) and Dietzenbacher, van der Linden, and Steenge (1993) that all linkages measured by the Leontief inverse can only be backward linkages, then this leaves the Ghosh price model as the only reasonable input-output structure to measure forward linkages. That is, those who accept this interpretation of the Ghosh model also accept the premise that primary-input price sensitivities are transmitted in a downstream fashion, that is, forwardly. Similarly, those who use the Leontief model concomitantly agree that final-demand changes can be met only by production from upstream industries, i.e., through backward linkages.

In addition, there is the joint stability issue (see footnote 21). We share the concern, first voiced by Cella (1984; also 1986, 1988), that conflicting requirements of constancy of both the **A** and **B** matrices must underpin any application that uses both models. But the issue is more gray than black or white. As an exercise in *ex post* analysis of economic structure, calculation of backward and forward linkage from an existing input-output data set for an economy—using, say, Cases 3b (Leontief) and 3a (Ghosh)—seems at first blush to avoid the Leontief-Ghosh joint stability issue, since it only employs rearrangements of an existing data set.[23] But a particular extraction, followed by calculation of an associated $\mathbf{\Delta x}$ vector, requires use of the appropriate Leontief or Ghosh inverse matrix and this means that we have bought into the underlying demand-pull or cost-push model. And many of these *ex post* studies ultimately come around to a "what if . . . ," *ex ante* focus wherein sectors (or regions) are categorized on the basis of both backward and forward linkage scores. Those with high readings on both measures are then singled out because of their (perceived) stimulative importance to the economy. And this too requires that the mechanisms of both Leontief and Ghosh models are at work simultaneously, thus raising the joint stability specter.

Table 1 summarizes the $\mathbf{\Delta x}$ results in terms of the partitioned difference matrix for the original seven hypothetical extractions from the **A** matrix (Leontief model) [given above in (9), (12), (15), (18), (21), (24) and (27)] along with the same seven possibilities that would occur in the **B** matrix (Ghosh model).[24] Outcomes on the unextracted (remaining) sectors are found by summing over the elements in $\mathbf{\Delta x_2}$.[25] For Case k in the Leontief quantity model, this means the sum of elements from the bottom *row* of the partitioned difference matrix, weighted by final demands—$\mathbf{i}'\mathbf{\Delta x_2} = \mathbf{i}'\mathbf{\Delta}^{\mathbf{kL}}_{\mathbf{21}}\mathbf{y_1} + \mathbf{i}'\mathbf{\Delta}^{\mathbf{kL}}_{\mathbf{22}}\mathbf{y_2}$. For the Ghosh price model, as in (33), and Case k, it is the sum of elements from the right-hand *column* of the partitioned difference matrix, weighted by value added—$\mathbf{\Delta x_2 i} = \mathbf{w}'_\mathbf{1}\mathbf{\Delta}^{\mathbf{kG}}_{\mathbf{12}}\mathbf{i} + \mathbf{w}'_\mathbf{2}\mathbf{\Delta}^{\mathbf{kG}}_{\mathbf{22}}\mathbf{i}$.

It is clear from Table 1 that Cases 1, 2a, 2b and 3b in the Leontief model generate identical results for $\mathbf{i}'\mathbf{\Delta x_2}$ and that Cases 1, 2a, 2c and 3a in the

[23] See, for example, Bulmer-Thomas (1982, Chap. 12), where both models are invoked for this kind of *ex post* analysis.

[24] In all of the submatrices for any of the cases in the Ghosh column in Table 1 it is also easily shown that $\mathbf{\Delta}^{\mathbf{kG}}_{ij} = (\hat{\mathbf{x}}_i)^{-1}\mathbf{\Delta}^{\mathbf{kL}}_{ij}(\hat{\mathbf{x}}_j)$. When a single sector is excluded, $\mathbf{H} = \mathbf{K} = s$ (a scalar) and so $\mathbf{\Delta}^{\mathbf{kG}}_{\mathbf{11}} = \mathbf{\Delta}^{\mathbf{kL}}_{\mathbf{11}}$ for $k = 1, 2\text{a}, \ldots, 3\text{c}$.

[25] While summing only over the elements of $\mathbf{\Delta x}_2$ often may be appropriate in an interindustry setting, where the usual story for the extraction is that one is measuring an industry's relative stimulative importance to the economy, this is not the case in an interregional setting. Here one is less often interested in analyzing the stimulative importance of a region but strictly the magnitude of its interregional linkages to the rest of the economy. Because of this the $\mathbf{\Delta x}$ results, rather than the $\mathbf{\Delta x}_2$ results, are typically used in this context unless one is applying Case 1. Similarly, if in an interindustry setting one is strictly interested in interindustry linkages as opposed to total linkages, the same observations apply.

Table 1: Partitioned Difference Matrices for Cases 1-3c ($k = 1, 2a, \ldots, 3c$)

Case	Structure of $\mathbf{A}^k$ or $\mathbf{B}^k$	Leontief Quantity Model	Ghosh Price Model
		$\begin{bmatrix} \boldsymbol{\Delta}_{11}^{kL} & \boldsymbol{\Delta}_{12}^{kL} \\ \boldsymbol{\Delta}_{21}^{kL} & \boldsymbol{\Delta}_{22}^{kL} \end{bmatrix}$	$\begin{bmatrix} \boldsymbol{\Delta}_{11}^{kG} & \boldsymbol{\Delta}_{12}^{kG} \\ \boldsymbol{\Delta}_{21}^{kG} & \boldsymbol{\Delta}_{22}^{kG} \end{bmatrix}$
1	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\mathbf{I} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\mathbf{I} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{bmatrix}$
2a	$\begin{bmatrix} \blacksquare & \mathbf{0} \\ \mathbf{0} & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\boldsymbol{\alpha}_{11} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\boldsymbol{\beta}_{11} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{21}\boldsymbol{\beta}_{22} \end{bmatrix}$
2b	$\begin{bmatrix} \mathbf{0} & \blacksquare \\ \mathbf{0} & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\mathbf{I} & (\mathbf{H}-\mathbf{I})\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\mathbf{I} & (\mathbf{K}-\mathbf{I})\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{bmatrix}$
2c	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \blacksquare & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\mathbf{I} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}(\mathbf{H}-\mathbf{I}) & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\mathbf{I} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}(\mathbf{K}-\mathbf{I}) & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{A}_{12}\boldsymbol{\beta}_{22} \end{bmatrix}$
3a	$\begin{bmatrix} \blacksquare & \mathbf{0} \\ \blacksquare & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\boldsymbol{\alpha}_{11} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}(\mathbf{H}-\boldsymbol{\alpha}_{11}) & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\boldsymbol{\beta}_{11} & \mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}(\mathbf{K}-\boldsymbol{\beta}_{11}) & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{bmatrix}$
3b	$\begin{bmatrix} \blacksquare & \blacksquare \\ \mathbf{0} & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\boldsymbol{\alpha}_{11} & (\mathbf{H}-\boldsymbol{\alpha}_{11})\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H} & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\boldsymbol{\beta}_{11} & (\mathbf{K}-\boldsymbol{\beta}_{11})\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K} & \boldsymbol{\beta}_{22}\mathbf{B}_{21}\mathbf{K}\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{bmatrix}$
3c	$\begin{bmatrix} \mathbf{0} & \blacksquare \\ \blacksquare & \blacksquare \end{bmatrix}$	$\begin{bmatrix} \mathbf{H}-\boldsymbol{\Phi} & (\mathbf{H}-\boldsymbol{\Phi})\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \boldsymbol{\alpha}_{22}\mathbf{A}_{21}(\mathbf{H}-\boldsymbol{\Phi}) & \boldsymbol{\alpha}_{22}\mathbf{A}_{21}(\mathbf{H}-\boldsymbol{\Phi})\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \end{bmatrix}$	$\begin{bmatrix} \mathbf{K}-\boldsymbol{\Psi} & (\mathbf{K}-\boldsymbol{\Psi})\mathbf{B}_{12}\boldsymbol{\beta}_{22} \\ \boldsymbol{\beta}_{22}\mathbf{B}_{21}(\mathbf{K}-\boldsymbol{\Psi}) & \boldsymbol{\beta}_{22}\mathbf{B}_{21}(\mathbf{K}-\boldsymbol{\Psi})\mathbf{B}_{12}\boldsymbol{\beta}_{22} \end{bmatrix}$

where $\boldsymbol{\alpha}_{ii} = (\mathbf{I}-\mathbf{A}_{ii})^{-1}$, $\boldsymbol{\beta}_{ii} = (\mathbf{I}-\mathbf{B}_{ii})^{-1} = (\hat{\mathbf{x}}_i)^{-1}\boldsymbol{\alpha}_{ii}(\hat{\mathbf{x}}_i)$, $\mathbf{H} = (\mathbf{I}-\mathbf{A}_{11}-\mathbf{A}_{12}\boldsymbol{\alpha}_{22}\mathbf{A}_{21})^{-1}$, $\mathbf{K} = (\mathbf{I}-\mathbf{B}_{11}-\mathbf{B}_{12}\boldsymbol{\beta}_{22}\mathbf{B}_{21})^{-1} = (\hat{\mathbf{x}}_1)^{-1}\mathbf{H}(\hat{\mathbf{x}}_1)$, $\boldsymbol{\Phi} = (\mathbf{I}-\mathbf{A}_{12}\boldsymbol{\alpha}_{22}\mathbf{A}_{21})^{-1}$, and $\boldsymbol{\Psi} = (\mathbf{I}-\mathbf{B}_{12}\boldsymbol{\beta}_{22}\mathbf{B}_{21})^{-1} = (\hat{\mathbf{x}}_1)^{-1}\boldsymbol{\Phi}(\hat{\mathbf{x}}_1)$.

Ghosh model produce identical results for $\mathbf{\Delta x_2 i}$. Since four of the seven possible extraction structures will generate identical results—for the Leontief quantity model and for the Ghosh price model—it may be unnecessary to quibble about the relative plausibilities of the various extractions.

6. Some empirical results

6.1 U.S. 1992 Data

We use data from a seven-sector version of the 1992 U.S. economy to illustrate these alternative measures.[26] Results, by industry, for each of the seven possible extractions from the Leontief quantity model are shown in Panel A of Table 2. For example, the first entry in column 2, \$316,070 (million), represents the value of the decrease in output throughout the entire U.S. economy when Agriculture (row 1) is extracted in the style of Case 2a (column 2); it is $\mathbf{i}'\mathbf{\Delta x^{2a}}$, from (12). Similarly, the fifth entry in column 6, \$651,140 (million), is $\mathbf{i}'\mathbf{\Delta x^{3b}}$ from (24), now with Trade and Transportation as the sector extracted in the manner of Case 3b [subscript 1 in (24)] and with the rest of the U.S. economy comprised of sectors 1–4, 6 and 7 [subscript 2 in (24)].

One way of putting these numbers into perspective is to normalize by the total pre-extraction output ($\mathbf{i}'\mathbf{x}$). Multiplied by 100, this indicates the percentage decrease in economy-wide output caused by the extraction. This is one plausible measure of the "importance" of the (extracted) sector to the economy; those figures, derived from Panel A, are shown in Panel B (as "Measure I") of Table 2. Looking again at Agriculture and Case 2a, we find that total output throughout the economy would decrease by almost three percent (2.92) in the absence of that sector; for Trade and Transportation and Case 3b, the decrease would be six percent.

While the numbers in either of the panels (A or B) in Table 2 are sometimes quite sensitive to how the extraction is carried out (compare, for example, Case 2a with 3c in any row), the *rankings* of sectors are quite stable. In Panel B, Cases 1, 2a, 2c and 3a give identical rankings to all sectors. The three most important sectors, measured by the effect of their removal, are Manufacturing, Services, and Trade and Transportation, in that order—and this ranking is constant across all seven cases. In that sense, if one is interested in using the $\mathbf{i}'\mathbf{\Delta x}$ results

[26] The seven-sector data are shown in Appendix B. They are from U.S. Bureau of Economic Analysis (1999). Aggregation from 94-sector to seven-sector detail was performed by Alexandru Voicu at the Center for Urban Policy Research, Rutgers University. The industry-by-industry accounts were developed following techniques described in Jackson (1998).

Table 2: Results for Leontief Extractions, using $\mathbf{i}'\Delta\mathbf{x}$

Sector	Case 1	Case 2a	Case 2b	Case 2c	Case 3a	Case 3b	Case 3c
A. Absolute Effect (millions of dollars)							
Agriculture	330,855	316,070	199,916	301,079	277,412	146,076	88,349
Mining	223,594	221,427	102,593	215,523	212,219	83,376	29,701
Construction	776,102	775,647	624,398	304,469	303,602	623,810	1,145
Manufacturing	2,528,852	1,891,051	2,018,767	2,015,425	1,153,361	1,158,163	1,248,141
Trade & Trans.	1,155,893	1,089,534	746,382	758,745	664,375	651,140	140,707
Services	2,524,714	1,746,394	1,931,384	1,992,564	1,062,802	984,212	1,192,086
Other	181,394	178,523	111,186	82,612	79,407	108,077	3,476
B. Relative Effect in (%) (Measure I)							
Agriculture	3.06	2.92	1.85	2.78	2.56	1.35	0.82
Mining	2.07	2.05	0.95	1.99	1.96	0.77	0.27
Construction	7.17	7.17	5.77	2.81	2.81	5.76	0.01
Manufacturing	23.37	17.47	18.65	18.62	10.66	10.70	11.53
Trade & Trans.	10.68	10.07	6.90	7.01	6.14	6.02	1.30
Services	23.33	16.14	17.85	18.41	9.82	9.09	11.01
Other	1.68	1.65	1.03	0.76	0.73	1.00	0.03
C. Relative Effect in (%) (Measure II)							
Agriculture	−70	−64	−76	−63	−48	−73	−77
Mining	−80	−75	−87	−73	−60	−84	−92
Construction	−30	−13	−24	−62	−43	16	−100
Manufacturing	129	113	146	149	115	116	223
Trade & Trans.	5	23	−9	−6	24	21	−64
Services	129	97	136	146	98	83	209
Other	−84	−80	−86	−90	−85	−80	−99

to identify the one or two most important sectors in an economy, it may not particularly matter what extraction scenario is used.[27]

We look at one further (and final) representation of the absolute results in Panel A. Denote the elements in that table as d_{ij}. For each column, j, find the average of the values in that column, subtract it from each element in the column, and then divide by that average, giving $d^*_{ij} = (d_{ij} - avg_j)/avg_j$. The average value of the elements d^*_{ij} in column j is zero, and $d^*_{ij} \times 100$ represents the percentage that the sector in row is above (if positive) or below (if negative) the average for that column. These figures are given in Panel C ("Measure II"). The rankings of sectors, for any given extraction method, are the same as for Measure I; the positive and negative terms may simply help in visualization of the results, since positive elements accompany sectors that are of above-average importance and negative entries are associated with sectors that have below-average importance in the economy.

As noted earlier, it can reasonably be argued that $\mathbf{i}'\mathbf{\Delta x_2}$ is a more appropriate measure of an extracted sector's importance—the output loss that is experienced by the *remaining* sectors after extracting one (or more) sectors. Table 3 contains results that parallel those in Table 2 for this alternative measure.

As would be expected from the structure of the partitioned matrices in the Leontief model column of Table 1, the $\mathbf{i}'\mathbf{\Delta x_2}$ results in Panel A of Table 3 are identical for Cases 1, 2a, 2b and 3b. Naturally, the dollar amounts are smaller than those in Table 2. The Measure I normalization in Panel B is now created through division by the original total output summed over the non-extracted sectors only. For example, the entries in row 4 of Panel A in Table 3 are divided by $\sum_{i \neq 4} x_i (= \mathbf{i}'\mathbf{x} - x_4)$—and multiplied by 100—to create row 4 in Panel B. This normalization may (reasonably) alter the rankings from those in Panel A, because for each sector the absolute effects are now related to a different denominator. For example, under Case 1, extraction of Manufacturing would cause the largest decrease in dollar value of output of the remaining sectors, while extraction of Services would result in the second largest decrease. But relative to the total pre-extraction output of unextracted sectors, the effect is largest in Services (13.34%) and second-largest in Manufacturing (13.18%). By contrast, Panel C rankings will remain the same as those in Panel A. The rankings of the three most important sectors using Measure II (Panel C) are

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[27] Groenewold, Hagger, and Madden (1993) use several methods of extracting an industry from a region (although not in the context of a partitioned matrix framework). Among others, they essentially use Cases 1, 2c, and 2b (or 3b, it is not quite clear) in a Leontief framework and note several "high correlations" in rankings. This is not surprising, given the results here and in Table 1.

Table 3: Results for Leontief Extractions, using $\mathbf{i}'\Delta\mathbf{x}_2$

Sector	Case 1	Case 2a	Case 2b	Case 2c	Case 3a	Case 3b	Case 3c
A. Absolute Effect (millions of dollars)							
Agriculture	142,763	142,763	142,763	112,987	104,105	142,763	33,155
Mining	82,254	82,254	82,254	74,183	73,046	82,254	10,223
Construction	616,484	616,484	616,484	144,851	144,439	616,484	545
Manufacturing	1,037,733	1,037,733	1,037,733	524,305	300,042	1,037,733	324,699
Trade & Trans.	622,359	622,359	622,359	225,211	197,200	622,359	41,765
Services	856,702	856,702	856,702	324,551	173,111	856,702	194,168
Other	107,416	107,416	107,416	8,635	8,300	107,416	363
B. Relative Effect in (%) (Measure I)							
Agriculture	1.35	1.35	1.35	1.07	0.98	1.35	0.31
Mining	0.77	0.77	0.77	0.70	0.68	0.77	0.10
Construction	6.08	6.08	6.08	1.43	1.42	6.08	0.01
Manufacturing	13.18	13.18	13.18	6.66	3.81	13.18	4.13
Trade & Trans.	6.66	6.66	6.66	2.41	2.11	6.66	0.45
Services	13.34	13.34	13.34	5.06	2.70	13.34	3.02
Other	1.08	1.08	1.08	0.09	0.08	1.08	0.00
C. Relative Effect in (%) (Measure II)							
Agriculture	−71	−71	−71	−44	−27	−71	−62
Mining	−83	−83	−83	−63	−49	−83	−88
Construction	25	25	25	−28	1	25	−99
Manufacturing	110	110	110	159	110	110	276
Trade & Trans.	26	26	26	11	38	26	−52
Services	73	73	73	61	21	73	125
Other	−78	−78	−78	−96	−94	−78	−100

identical to those in Table 2, with the exception of Case 3a, in which the second- and third-highest ranking sectors are interchanged.

Results for extractions in the Ghosh price model are shown in Tables 4 and 5, for $\mathbf{\Delta xi}$ and $\mathbf{\Delta x_2 i}$, respectively. Again, as would be expected from the partitioned matrices in the Ghosh model column of Table 1, the $\mathbf{\Delta x_2 i}$ results in Panel A of Table 5 are identical for Cases 1, 2a, 2c and 3a. And, as in Table 3, the normalizations in Panel B of Table 5 may alter sectoral rankings, compared to those in Panels A and C (as is true, for example, in Cases 2b, 3b and 3c).

6.2 Additional Empirical Results

Case 1 and Case 2a extractions using 23-sector U.S. data were calculated in the mid-1990s as part of a graduate seminar project overseen by one of us (Miller). These data covered the years 1947, 1958, 1963, 1967, 1972, 1977, 1982 and 1987.[28] Results are compared in Table 6. The top panel in this table indicates the generally quite consistent rankings of the top three sectors, as measured by $\mathbf{i}'\mathbf{\Delta x}$ from the Leontief quantity model, using these two extraction approaches. The bottom panel contains Spearman's rank-correlation coefficient between the two extraction methods for each of the years. Again, it is clear that the rankings are highly correlated. A similar high rank correlation for these two extraction approaches was found by Cochrane (1990) using 21-sector input-output data for 1980 for the Luwu region of Sulawesi in Indonesia.

7. Conclusions

It is appropriate to reflect on the motivation for linkage measures. Initially, quantification of forward and backward linkages was driven by economic development interests (Hirschman). The object was to identify key sectors in less-developed (and later, in regional) economies, because it was believed that encouragement of (concentration of scarce resources in) those sectors would lead to maximum economy-wide benefits, thereby stimulating economic development. For this purpose, evaluation of a sector's *total* linkage seems to us to be the appropriate measure. In that case, we believe the original hypothetical extraction approach (Paelinck, de Caevel, and Degueldre; Strassert;

[28] The 1947-1977 data are in Miller and Blair, 1985, Appendix B. Those for 1982 and 1987 were produced in 1994 by the Social Science Data Center, University of Pennsylvania, under the direction of Dr. Janusz Szyrmer.

Table 4: Results for Ghosh Extractions, using $\mathbf{i}'\Delta\mathbf{x}$

Sector	Case 1	Case 2a	Case 2b	Case 2c	Case 3a	Case 3b	Case 3c
A. Absolute Effect (millions of dollars)							
Agriculture	380,489	350,781	278,893	299,579	245,739	218,883	111,495
Mining	283,268	270,952	163,929	234,301	215,084	134,797	46,070
Construction	598,618	598,327	475,884	259,115	258,528	475,486	823
Manufacturing	2,429,737	1,847,784	2,063,732	1,791,792	931,188	1,321,910	1,177,123
Trade & Trans.	1,232,390	1,163,033	737,934	865,381	770,139	633,703	148,692
Services	2,598,701	1,780,354	1,899,003	2,146,021	1,198,849	881,534	1,240,733
Other	184,036	181,130	71,853	124,271	121,163	68,567	3,521
B. Relative Effect in (%) (Measure I)							
Agriculture	3.52	3.24	2.58	2.77	2.27	2.02	1.03
Mining	2.62	2.50	1.51	2.16	1.99	1.25	0.43
Construction	5.53	5.53	4.40	2.39	2.39	4.39	0.01
Manufacturing	22.45	17.07	19.07	16.56	8.60	12.21	10.88
Trade & Trans.	11.39	10.75	6.82	8.00	7.12	5.86	1.37
Services	24.01	16.45	17.55	19.83	11.08	8.15	11.46
Other	1.70	1.67	0.66	1.15	1.12	0.63	0.03
C. Relative Effect in (%) (Measure II)							
Agriculture	−65	−60	−66	−63	−54	−59	−71
Mining	−74	−69	−80	−71	−60	−75	−88
Construction	−46	−32	−41	−68	−52	−11	−100
Manufacturing	121	109	154	119	74	148	202
Trade & Trans.	12	31	−9	6	44	19	−62
Services	136	101	134	163	124	65	218
Other	−83	−80	−91	−85	−77	−87	−99

Table 5: Results for Ghosh Extractions, using $\mathbf{i}'\Delta\mathbf{x}_2$

Sector	Case 1	Case 2a	Case 2b	Case 2c	Case 3a	Case 3b	Case 3c
A. Absolute Effect (millions of dollars)							
Agriculture	242,426	242,426	140,830	242,426	242,426	110,527	56,300
Mining	213,962	213,962	94,623	213,962	213,962	77,807	26,598
Construction	251,201	251,201	128,467	251,201	251,201	128,359	222
Manufacturing	810,758	810,758	444,753	810,758	810,758	284,883	253,680
Trade & Trans.	741,358	741,358	246,902	741,358	741,358	212,028	49,750
Services	1,071,339	1,071,339	371,641	1,071,339	1,071,339	172,519	242,815
Other	120,502	120,502	8,319	120,502	120,502	7,939	408
B. Relative Effect in (%) (Measure I)							
Agriculture	2.29	2.29	1.33	2.29	2.29	1.04	0.53
Mining	2.01	2.01	0.89	2.01	2.01	0.73	0.25
Construction	2.48	2.48	1.27	2.48	2.48	1.27	0.00
Manufacturing	10.30	10.30	5.65	10.30	10.30	3.62	3.22
Trade & Trans.	7.93	7.93	2.64	7.93	7.93	2.27	0.53
Services	16.69	16.69	5.79	16.69	16.69	2.69	3.78
Other	1.22	1.22	0.08	1.22	1.22	0.08	0.00
C. Relative Effect in (%) (Measure II)							
Agriculture	−51	−51	−31	−51	−51	−22	−37
Mining	−57	−57	−54	−57	−57	−45	−70
Construction	−49	−49	−37	−49	−49	−10	−100
Manufacturing	64	64	117	64	64	101	182
Trade & Trans.	50	50	20	50	50	49	−45
Services	117	117	81	117	117	21	170
Other	−76	−76	−96	−76	−76	−94	−100

Table 6: Results for U. S. 23-Sector Models

Orderings of Top Three Sectors*
(x,y,z indicates first-, second- and third-ranked sectors)

Year	1947	1958	1963	1967	1972	1977	1982	1987
Case 1	6,21,5	21,5,6	5,21,6	21,5,6	5,19,6	5,19,21	19,5,21	19,5,21
Case 2a	21,6,1	21,19,5	21,19,5	21,19,5	21,19,5	19,21,5	21,19,5	21,19,5

Spearman's Rank-Correlation Coefficient between Cases 1 and 2a Rankings

Year	1947	1958	1963	1967	1972	1977	1982	1987
Coefficient	0.97	0.96	0.96	0.98	0.97	0.94	0.92	0.92

* Sectors: 1 (Agriculture, Forestry, Fishing), 5 (Construction), 6 (Food, Feed, Tobacco Products), 19 (Trade, Transportation), 21 (Services, excluding Electric, Gas and Sanitary)

Schultz; as in Case 1, Leontief model) is totally adequate—Mellar and Marfán and other modifications notwithstanding.[29]

Moreover, if one accepts the effect on the remaining sectors only, $\mathbf{i}'\Delta\mathbf{x}_2$, as the appropriate measure of impact (of an extraction), then we have shown that several alternative extraction structures produce identical results. Even using the entire $\mathbf{i}'\Delta\mathbf{x}$ measure appears to generate the same rankings of sectoral importance in many cases (at least this is suggested by our numerical illustrations). We see little point in worrying about how to decompose a total linkage measure into forward and backward components when, for key sector identification, it is a combined measure that is needed.

Perhaps for cross-economy comparisons of economic structure (across regions in a multiregional economy, across countries in the EEC, etc.) there is a place for separate backward and forward linkage indicators. In that event, we believe that the vector generated by Case 3b from the Leontief model is the appropriate backward linkage measure. For forward linkages, we vote for Case 3a, this time from the Ghosh model. As noted in Section 5, we believe that there are problems in accepting Cella's decomposition of the Leontief inverse into forward and backward linkage, invoking prior statements by Guccione and Gillen (1986) and Dietzenbacher, van der Linden, and Steenge (1993) that the Leontief

[29] Dietzenbacher and van der Linden (1997, p. 236) criticize Strassert's hypothetical extraction approach on two grounds: (1) it "…does not distinguish the total linkages into backward and forward linkages…" and (2) "…simply scrapping an entire sector from the economy seems to be rather excessive." We see nothing wrong with a single, total linkage measure, and we do not find Strassert's extraction excessive.

Table 7: Sectoral Rankings across Total Linkage Measures (1992 U.S. data; see Appendix B)

	Backward Linkage Leontief Model				Forward Linkage							
					Leontief Model				Ghosh Model			
	$\mathbf{i}'\Delta\mathbf{x}$		$\mathbf{i}'\Delta\mathbf{x}_2$		$\mathbf{i}'\Delta\mathbf{x}$		$\mathbf{i}'\Delta\mathbf{x}_2$		$\mathbf{i}'\Delta\mathbf{x}$		$\mathbf{i}'\Delta\mathbf{x}_2$	
	Case		Case*		Case		Case		Case		Case*	
Sector	2b	3b	2b	3b	2c	3a	2c	3a	2c	3a	2c	3a
1	5	5	5	5	5	5	5	5	4	4	5	5
2	7	7	7	7	6	6	6	6	6	6	6	6
3	4	4	4	4	4	4	4	4	5	5	4	4
4	1	1	2	2	1	1	1	1	2	2	2	2
5	3	3	3	3	3	3	3	3	3	3	3	3
6	2	2	1	1	2	2	2	2	1	1	1	1
7	6	6	6	6	7	7	7	7	7	7	7	7

* Rankings in these two columns will be identical, by definition.

inverse can only measure backward linkages. This leads us to support use of the Ghosh model to measure forward linkages.

If one prefers to extract *intrasectoral* connections as well in assessing linkage, this would mean using Case 2b (not 3b) with the Leontief model for backward linkages and Case 2c (not 3a) with the Ghosh model for forward linkages. Table 7 indicates the similarity of rankings under these various alternatives, for our numerical illustration with U.S. 1992 data. In particular, the same sectors constitute the top three (admittedly with some differences in orderings) across all indicators in the table. Further, we conjecture that differences between members of these pairs of alternative measures would diminish as the number of industries depicted in the model increases.

The results in any post-extraction $\Delta\mathbf{x}_2$ vector are of course sector-specific. Suppose that sector 1 is extracted; then $\Delta\mathbf{x}_2 = \begin{bmatrix} \mathbf{x}_{22} \\ \vdots \\ \mathbf{x}_{2n} \end{bmatrix}$, where $\Delta\mathbf{x}_{2j}$ is the decrease in sector *j*'s output when sector 1 is removed. There may be studies for which this level of detail is appropriate; it is of course lost in forming an aggregate measure of sector 1's linkage (as in $\mathbf{i}'\Delta\mathbf{x}_2$). With multiregional input-output data sets, there is both sectoral and spatial detail in a $\Delta\mathbf{x}_2$ vector—output decreases in all remaining sectors in each region. Alternative measures of either sectoral or spatial linkage (or both) emerge from different aggregations of the $\Delta\mathbf{x}_2$ vector.[30]

An important outcome of our investigation is that when $\Delta\mathbf{x}_2$—the total effect on the unextracted sectors—is used as the appropriate indicator of total linkage for the extracted sector, whether in the Leontief or the Ghosh model, four of the seven possible measures lead to exactly the same numerical result. This suggests that arguments about the virtues or vices of alternative extraction methods may be both unproductive and unnecessary.

We use a seven-sector representation of the 1992 U.S. economy to illustrate the fourteen alternative measures. We find that the *rankings* were quite stable across all seven measures for a particular model (Leontief or Ghosh) as well as for the same measure across the two models. Similar stability of rankings was found across Leontief Cases 1 and 2a for a 23-sector version of the U.S. economy for each of eight years. And for studies in which one wants to identify the top one or two sectors, this means, again, that the particular extraction structure and model employed may be relatively unimportant. We note, however, that much of the stability may result from the high level of aggregation (especially the seven-sector results) and that a more uniform distribution of industry sizes could yield very different results. Thus, more empirical work in this regard is appropriate and would be welcome.

Appendix A: A note on interregional feedbacks, linkages and normalizations

In Miller (1966, 1969), a measure of interregional feedback effects was proposed in order to investigate the inaccuracy that might result from using a single-region input-output model in an inherently interconnected many-region economy. The scenario was: Assume a change in demand in a region—for example, commercial airplane exports from the State of Washington. How much is the statewide impact of this exogenous demand change underestimated if a one-region (Washington) input-output model is used instead of an interconnected "two-region" model (Washington/Rest of the U.S.)? In "spatial extraction" terms, let subscript "1" denote the rest of the U.S. and "2" denote Washington; $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \hdashline \mathbf{x}_2 \end{bmatrix}$ specifies outputs in the rest of the U.S. ($\mathbf{x}_1$) and in Washington ($\mathbf{x}_2$) in a two-region input-output model. Let $\mathbf{x}^1 = \begin{bmatrix} \mathbf{x}_1^1 \\ \hdashline \mathbf{x}_2^1 \end{bmatrix}$ denote

[30] See, for example, Dietzenbacher and van der Linden (1997). Earlier examples of these kinds of measures applied to spatial linkage (but invoking both Leontief and Ghosh models simultaneously) can be found in Miller and Blair (1988) and Blair and Miller (1990).

these outputs when region 1 is completely extracted (the "Washington-alone" model). Using (2), letting $\mathbf{y_2}$ = the *change* in final demand in Washington (the new airliner exports) and $\mathbf{y_1} = \mathbf{0}$ (the exogenous final demand shock occurs only in Washington),

$$\mathbf{x_2} = \boldsymbol{\alpha}_{22}(\mathbf{I} + \mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22})\mathbf{y_2}$$

and since

$$\mathbf{x}_2^1 = (\mathbf{I} - \mathbf{A}_{22})^{-1}\mathbf{y_2} = \boldsymbol{\alpha}_{22}\mathbf{y}_2$$

the difference in statewide output

$$\mathbf{i}'(\mathbf{x_2} - \mathbf{x}_2^1) = \mathbf{i}'(\Delta\mathbf{x}_2^1) = \mathbf{i}'(\boldsymbol{\alpha}_{22}\mathbf{A}_{21}\mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22})\mathbf{y_2}$$

[as in (9)], indicates the importance of interregional feedback effects to Washington when assessing the impact of a change in $\mathbf{y_2}$ only. This was the indicator used in Miller (1966, 1969).[31] From this point of view, it is logical to create a normalization like $(\mathbf{i}'\Delta\mathbf{x_2}/\mathbf{i}'\mathbf{x_2}) \times 100$—the percentage "error" that would occur in estimating total state output (summed over all sectors) when the full two-region model is ignored. This provides an answer to a question like: how important for the state (not extracted) are economic connections to the rest of the U.S. (extracted)?[32]

When extraction approaches are used to measure backward (or forward) linkages, the normalization is often carried out in the opposite way, namely through division of the output difference by the output of the sector (region) that *is* extracted (as in Dietzenbacher, van der Linden, and Steenge, 1993). This is also reasonable; it simply reflects a different focus. In our example, it would provide an answer to a question like: how important for the rest of the U.S. economy (extracted) are connections to the state (not extracted)?

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[31] In that work, the measure was the simpler expression

$$\Delta\mathbf{x_2} = [(\mathbf{I} - \mathbf{A}_{22} - \mathbf{A}_{21}\boldsymbol{\alpha}_{11}\mathbf{A}_{12})^{-1} - (\mathbf{I} - \mathbf{A}_{22})^{-1}]\mathbf{y_2}$$

This came from an alternative and equally valid expression for the partitioned Leontief inverse, namely

$$\mathbf{L} = \left[\begin{array}{c|c} \boldsymbol{\alpha}_{11}(\mathbf{I} + \mathbf{A}_{12}\mathbf{H}^*\mathbf{A}_{21}\boldsymbol{\alpha}_{11}) & \boldsymbol{\alpha}_{11}\mathbf{A}_{12}\mathbf{H}^* \\ \hline \mathbf{H}^*\mathbf{A}_{21}\boldsymbol{\alpha}_{11} & \mathbf{H}^* \end{array}\right]$$

where $\mathbf{H}^* = (\mathbf{I} - \mathbf{A}_{22} - \mathbf{A}_{21}\boldsymbol{\alpha}_{11}\mathbf{A}_{12})^{-1}$ and $\boldsymbol{\alpha}_{11} = (\mathbf{I} - \mathbf{A}_{11})^{-1}$. A "symmetric" form of this partitioned Leontief inverse could also be used; it is

$$\mathbf{L} = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{H}\mathbf{A}_{12}\boldsymbol{\alpha}_{22} \\ \hline \mathbf{H}^*\mathbf{A}_{21}\boldsymbol{\alpha}_{11} & \mathbf{H}^* \end{array}\right]$$

(See, for example, Sonis, Hewings, and Miyazawa, 1997, and their citations to its origins.).

[32] See Round (2001) for a review of the interregional feedback literature.

Appendix B: Data

1992 Seven-Sector U.S. Input-Output Transactions Table (Millions of dollars)

	Agriculture	Mining	Construction	Manufacturing	Trade & Trans.	Services	Other	Final Demand	Total Output
Agriculture	54,601	43	3,932	116,326	903	11,974	312	49,570	237,662
Mining	450	19,355	5,338	69,177	1,517	42,797	2,707	15,377	156,717
Construction	2,895	2,670	594	18,133	11,502	102,672	21,152	519,712	679,330
Manufacturing	36,114	10,226	184,624	897,216	96,187	255,963	10,791	1,460,183	2,951,303
Trade & Trans.	17,968	5,372	68,420	242,656	97,138	96,463	5,517	940,859	1,474,393
Services	25,179	30,960	83,185	262,308	271,016	975,420	19,945	2,734,957	4,402,970
Other	856	681	1,323	13,164	12,770	42,073	3,111	846,294	920,272
Value Added	99,599	87,410	331,913	1,332,324	983,361	2,875,608	856,738		

1992 Seven-Sector U.S. Direct Requirements Matrix

0.22974	0.00027	0.00579	0.03942	0.00061	0.00272	0.00034
0.00189	0.12350	0.00786	0.02344	0.00103	0.00972	0.00294
0.01218	0.01704	0.00087	0.00614	0.00780	0.02332	0.02298
0.15196	0.06525	0.27177	0.30401	0.06524	0.05813	0.01173
0.07560	0.03428	0.10072	0.08222	0.06588	0.02191	0.00599
0.10594	0.19755	0.12245	0.08888	0.18382	0.22154	0.02167
0.00360	0.00435	0.00195	0.00446	0.00866	0.00956	0.00338

1992 Seven-Sector U.S. Direct Allocations Matrix

0.22974	0.00018	0.01654	0.48946	0.00380	0.05038	0.00131
0.00287	0.12350	0.03406	0.44141	0.00968	0.27308	0.01727
0.00426	0.00393	0.00087	0.02669	0.01693	0.15114	0.03114
0.01224	0.00346	0.06256	0.30401	0.03259	0.08673	0.00366
0.01219	0.00364	0.04641	0.16458	0.06588	0.06543	0.00374
0.00572	0.00703	0.01889	0.05958	0.06155	0.22154	0.00453
0.00093	0.00074	0.00144	0.01430	0.01388	0.04572	0.00338

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