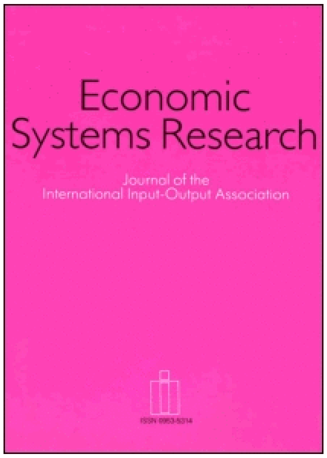

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A Revision of the Tolerable Limits Approach: Searching for the Important Coefficients

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A Revision of the Tolerable Limits Approach: Searching for the Important Coefficients

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ABSTRACT *A wide range of approaches are available for classifying coefficients according to their importance to an economy. The 'tolerable limits' approach is one that has been extensively written about. Nevertheless, it seems unsuitable for assessing the overall importance of a coefficient to an economy, but instead appears to be rather well suited for determining how much a selling sector depends upon its customers. We therefore suggest two alternative approaches for measuring a sector's importance to an economy. The first is an application of the concept of elasticity based on Sherman and Morrison's (1950) formula. The second approach applies linear programming. We compare these various alternatives using the domestic IO tables of eight European countries.*

KEY WORDS: Tolerable limits, elasticity, linear programming, important coefficients

1. Introduction

It has been more than 50 years since Sherman and Morrison (1950) established a formula for estimating the effects on an inverse matrix of a change in an element of the original matrix. This formula enables input–output analysts to assess the consequences in output of changes in direct requirements coefficients. In this context, one of its first applications identified the set of economic transactions that most influence the output of the economy. The input–output coefficients underlying the most critical transactions have

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generally been called 'important coefficients' (ICs). The ability to identify ICs opens up a wide range of applications within the input–output framework, which can be classified into two categories.

In the first, identifying ICs enables builders of income accounts to be more able to direct statistical resources toward portions of tables so as to create more accurate input–output accounts (see Bródy, 1970; Bullard and Sebald, 1977; Dietzenbacher, 1988; and Xu and Madden, 1991, among others).[1] The second relates to structural change analysis – more specifically to the framework of economic integration and economic development. The level of economic integration in a country is characterized by its concentration of ICs, which determine the fundamental economic structure of such a country (see Simpson and Tsukui, 1965; Jensen *et al.*, 1988). A country with a relatively high number of ICs spread across the network of intersectoral relationships is likely to be highly integrated. In contrast, few ICs implies little integration, since the country's economy must necessarily be restricted to a limited set of isolated sectors. Thus, its pattern of economic development is likely to be unbalanced and depending heavily upon the relative productivity of those sectors. From a public policy perspective, a general level of economic integration is a good thing, since it means that most sectoral policies will spread their effects widely within the productive network: thus contributing to balanced growth of the entire economy (see Aroche Reyes, 2002). Hence, the location and frequency of ICs can be critical in determining the direction and nature of economic policies.

Of course, much of the above discussion hinges on what is meant by 'important' in terms of ICs. Different concepts of 'importance' can lead to very different evaluations of ICs, and therefore of their location and frequency within an input–output table. In the case of economy-wide structural analysis, the importance of a coefficient should be related to its capacity to influence the output of the economic system as a whole. Thus defined, an economy's 'connectedness' (Szyrmer, 1985) or integration level is considered as 'high' if it contains a large number of ICs that are highly capable of affecting the economy's gross output. In addition, if we look at the notion of welfare growth instead of economic growth from a political point of view, establishing the evaluation of ICs in terms of variables different from the country's gross output – such as its employment level, energy usage, or environment quality – could prove very useful in certain applications.

Owing to the very wide range of applications in which this important concept can be used, it will be a focus of this paper. That is, in this paper, we use the premise that it is preferable to discover coefficients with a high capacity for diffusing sectoral-based actions, since they are best poised to yield improvements in societal welfare.

There are many approaches that classify coefficients according to their importance. The 'tolerable limits' (TL) approach is perhaps the one that is most extensively published. The tenets of TL, however, do not match our definition of importance. Instead, it identifies for each coefficient the maximum amount by which one could alter the coefficient ('tolerable limit') and still not change output in a sector by more than some prespecified percentage p (usually 1%) in the output.

As will be verified later, the 'most affected' sector identified by TL is the selling sector in the specified coefficient. Thus, TL measures 'importance' as the greatest extent to which a buying sector depends upon a selling sector. Therefore, TL may well have a role in building statistics tables, since it identifies coefficients that could create the greatest estimation 'errors' in a selling sector. However, TL does not appear to be appropriate for analyzing economic growth and integration, since it does not offer any assurance that the coefficient identified as important plays a major economic role. In other words, an important coefficient identified via the TL approach does not necessarily yield the greatest amount of economy-wide interdependence among the set of direct coefficients in the table. Rather, it simply identifies the coefficient that induces the most interdependence among any two industries in an economy.

Based upon the above discussion, the point of this paper will be to determine whether importance, such as that imbedded in the TL formula is in fact sufficiently parallel in nature to the more general concept of 'importance' based upon economy-wide welfare. With this in mind, we present a series of alternative approaches to evaluate the importance of coefficients from the more global perspective, and later compare empirical results from them to results from the TL approach using the same set of input–output tables.

The paper proceeds in the following way. The next section gives a backdrop to the techniques that identify ICs and provides the foundations of the TL algorithm. The section after shows how the TL formula can be interpreted as an elasticity, which leads to several other extensions of Sherman and Morrison's (1950) formula. The fourth section focuses upon an alternative approach for the identification of the IC based on linear programming. The fifth section carries out a comparison among the different approaches pointed out in earlier sections to verify the stability of the coefficient order regardless of the underlying definition of 'importance'. Finally, conclusions are presented in the sixth section.

2. The 'Tolerable Limits' Approach for Locating the Important Coefficients

As previously explained, the IC of the economic system can be defined as those with minimal variations that involve great changes in the output. According to the version of TL technique developed by Schintke and Stäglin (1985, 1988), the algorithm identifies, for each coefficient, the maximum variation margin that does not lead to changes greater than a p percentage in the output of at least one activity branch. This margin is calculated as (see Schintke and Stäglin, 1985, p. 130):

$$r_{kl} = \frac{p}{a_{kl}\left(\dfrac{b_{lk}p}{100} + \max\limits_{i=1,\dots,n} \dfrac{b_{ik}x_l}{x_i}\right)} = \frac{p}{a_{kl}\left(\dfrac{b_{lk}p}{100} + \dfrac{b_{kk}x_l}{x_k}\right)} \tag{1}$$

where: a_{kl} = input coefficient, calculated as the value of deliveries from activity k to activity l per unit of output of l; x_l = output of activity l; b_{kl} = element (k, l) of the Leontief inverse $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$.

The crucial part in this equation is $\text{Max}_i(b_{ik}/x_i) = (b_{kk}/x_k)$. A simple proof (to our knowledge not made explicit previously) follows using Metzler's theorem (Metzler, 1945; see also Sierksma, 1979). Let matrix $\mathbf{A}$ be non-negative and irreducible, and let its column sums be no larger than one. Then, Metzler's theorem states that $\text{Max}_i b_{ki} = b_{kk}$, i.e. the diagonal element of the Leontief inverse $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ is the largest element in its row. Now consider an input–output table with intermediate deliveries given by z_{kl} and final demands by y_k. Input coefficients have been defined as $a_{kl} = z_{kl}/x_l$. Let us define the so-called output coefficients as $w_{kl} = z_{kl}/x_k$. The relationship between the input and output coefficients is given by $w_{kl} = a_{kl}x_l/x_k$, or in matrix notation by $\mathbf{W} = \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}$. Note that the output coefficient w_{kl} gives the fraction of output in activity

k that is delivered to activity l. Assuming that no final demand y_k is negative, this implies that no row sum of $\mathbf{W}$ (or column sum of its transposed $\mathbf{W}'$) is larger than one. This means that the conditions for Metzler's theorem hold for the transposed matrix $\mathbf{W}'$. Hence, the diagonal element of $(\mathbf{I} - \mathbf{W}')^{-1}$ is the largest in its row. Since the inverse of a transposed matrix is the transpose of the inverse, this implies that the diagonal element of $(\mathbf{I} - \mathbf{W})^{-1}$ is the largest in its column. Because $\mathbf{W} = \hat{\mathbf{x}}^{-1}\ \mathbf{A}\hat{\mathbf{x}}$, it immediately follows that $(\mathbf{I} - \mathbf{W})^{-1} = \hat{\mathbf{x}}^{-1}\ (\mathbf{I} - \mathbf{A})^{-1}\ \hat{\mathbf{x}} = \hat{\mathbf{x}}^{-1}\ \mathbf{B}\hat{\mathbf{x}}$. The diagonal element being the largest in its column, yields for column k that $\mathrm{Max}_i(b_{ik}\ \ x_k\ \ /x_i) = (b_{kk}\ \ x_k/x_k)$, which proves $\mathrm{Max}_i(b_{ik}/x_i) = (b_{kk}\ /x_k)$.

The TL formula in equation (1) is often used to locate the ICs. However, as suggested above, several criticisms can be levied against its application in certain situations. The first criticism relates to the ceteris paribus nature of the approach. That is, it assesses the importance of a given direct coefficient supposing only this one coefficient changes. Indeed, in the most practicable applications it would seem to be more desirable to assess the overall importance of a set of coefficients than of a single, isolated coefficient. Therefore, it would be an advantage if the approach was sufficiently flexible to allow one to consider the importance of joint coefficient changes. Such a generalization is especially important in at least two major applications: identifying key-sectors and identifying sectors as targets of further enhancement with primary data (the work of national statistical bureaus). In all cases, checking the importance of columns or rows of coefficients is more pragmatic than checking the importance of isolated coefficients (Lahr, 2001). In this regard, several general extensions of Sherman and Morrison (1950) have been developed to measure changes in a given row or column of a coefficients matrix (see Dwyer and Waugh, 1953; Evans, 1954; Maaβ, 1980; West, 1982; Schintke and Stäglin, 1988, among others).

Furthermore, the effect of a change in a given coefficient on an economy depends upon the extent to which the coefficient is the only one that varies or whether, instead, simultaneous changes take place in a set of coefficients since synergic effects can be caused through interaction among coefficients. Obviously, if such synergic effects associated with a given coefficient are considerable, its relative importance could be greatly enhanced. Therefore, these joint effects should be also considered.[2] For instance, when an innovation affects the technology of a given sector, we should consider the change of all the sector's entire column of coefficients rather than change in only one of them.

More fundamentally, however, in this paper we will focus upon a second criticism of TL – the nature of its concept of 'importance'. With this end in mind, we test whether the ordering of coefficients by their importance is significantly different when defined by means of the TL approach as opposed to another definition of 'importance' more suitable to the most practicable applications – the coefficient's (or set of coefficients') contribution to the overall welfare.

So why do we contend that TL's concept of importance is inadequate in assessing a direct coefficient's contribution to overall welfare? Schnabl (2003) provides a valuable starting point for the discussion. He shows how Maaβ's (1980) elasticity concept reinterprets the TL formula. More specifically, he shows how equation (1) can be reinterpreted as the inverse of the elasticity of the output of the most sensitive activity with regards to a given technical coefficient. This reinterpretation, which follows, allows us to build several extensions towards an approach that meets our perceived needs.

3. A Review of the 'Tolerable Limits' Approach, based on the 'Elasticity' Concept

Maaβ (1980) estimates the relative change of output x_i of activity i when coefficient a_{kl} is changed by a d percentage. This is essentially the elasticity of the output of activity i with respect to a_{kl}. His formula for calculating this elasticity is:

$$\varepsilon_{x_i a_{kl}}(d) = b_{ik} a_{kl} \frac{\sum_j b_{lj} y_j}{(1 - d a_{kl} b_{lk}) x_i} \tag{2}$$

where y_j is the final demand to activity j. Maaβ suggests the maximum elasticity is the measure of the importance of coefficient a_{kl}. When $i = k$, the maximum value is attained. Thus, equation (2) converts into the following expression:

$$\varepsilon^{\max}_{x_i a_{kl}}(d) = \max_i \varepsilon_{x_i a_{kl}}(d) = b_{kk} a_{kl} \frac{\sum_j b_{lj} y_j}{(1 - d a_{kl} b_{lk}) x_k} \tag{3}$$

Finally, considering $\Sigma_j b_{lj} y_j = x_l$ and $d < 1$ (so that $d a_{kl} b_{lk} \approx 0$), the expression (3) will be simplified as:

$$\varepsilon^{\max}_{x_i a_{kl}}(d) \approx b_{kk} a_{kl} \frac{x_l}{x_k} \tag{4}$$

Now, reflecting upon the TL formula (1), assume $p = 1$ (that is, a 1% of variation of the output). In this case, the left-hand term of the denominator of equation (1) practically vanishes, becoming:

$$r_{kl} \approx \frac{1}{a_{kl} b_{kk} x_l / x_k} \approx \frac{1}{\varepsilon^{\max}_{x_i a_{kl}}(d)} \tag{5}$$

Thus, the TL formula is equivalent to the inverse of the elasticity of the output for coefficient a_{kl}. For instance, consider coefficient a_{kl} with a tolerable limit of $r_{kl} = 20$. That means that if the coefficient changes by 20%, the output of an activity (activity k) increases by 1% (value of p). Owing to the linear nature of the assumptions in the input–output relations, we can also say that a change of 1% in the coefficient leads to a change of 0.05% in the output (the inverse of 20%). This is the elasticity of the output of the aforementioned activity with respect to the given coefficient a_{kl}. This reinterpretation facilitates reflection upon several questions, and helps to extend the elasticity concept in order to address them.

The first question reflected upon is related to the criterion used to measure the importance of coefficient a_{kl}. The TL formula proposes a maximum variation that does not lead to changes greater than a p percentage in the output of one activity branch. As we can verify in equation (5), this is equivalent to computing the inverse of Maaβ's (1980) maximum elasticity. Clearly, in both cases, the maximum value is reached when $i = k$. That is, the maximum is attained when the reference activity branch is activity k, i.e. the selling activity in the transaction involved by coefficient a_{kl}. Thus, it would appear that the TL formula assesses the dependence of a selling sector on the purchasing sector, with the two sectors being identified by the particular location of the a_{kl} within the input–output table.

Nevertheless, from an inter-connected economic perspective, we should clearly be interested in identifying the sectoral interconnections that maintain the largest potential for providing the greatest overall welfare. In other words, a_{kl} could be classified as being of TL importance, as long as its minimal variation creates great variation in the output of k. However, it may well be that activity k maintains a small share of an economy's overall activity. Thus, while coefficient a_{kl} may well be important to sector k, indeed even making sector j more important to k than it is to any other sector, a sizeable change in a_{kl} need not be important at all to the broader economy. That is, if a change in coefficient a_{kl} were effected, it need not be a change that was important to the economy as a whole.

Accordingly, we propose an alternative measure for evaluating the importance of a coefficient based on the concept of elasticity. This measure is based on a coefficient's capacity (elasticity) to change overall output. This elasticity would be expressed as:

$$\varepsilon_{x_\bullet a_{kl}} = \frac{\sum_{i=1}^{n} \Delta x_i \Big/ \sum_{i=1}^{n} x_i}{\Delta a_{kl}/a_{kl}} \tag{6}$$

Given $x_i = \Sigma_{j=1}^{n} b_{ij} y_j$, we know that the variation in the activity's output depends on the changes in the elements of the corresponding Leontief inverse matrix row. These changes are provided by Sherman and Morrison's (1950) formula, as follows:

$$\Delta b_{ij} = \frac{b_{ik} b_{lj} \Delta a_{kl}}{1 - b_{lk} \Delta a_{kl}} \tag{7}$$

Thus, integrating equation (7) into equation (6) yields:

$$\varepsilon_{x_\bullet a_{kl}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{b_{ik} b_{lj} \Delta a_{kl}}{1 - b_{lk} \Delta a_{kl}} y_j \Big/ \sum_{i=1}^{n} x_i}{\Delta a_{kl}/a_{kl}} \tag{8}$$

Considering $\Delta a_{kl} = d a_{kl}$, where d is the percentage of variation of a_{kl},

$$\varepsilon_{x_\bullet a_{kl}}(d) = \frac{\frac{a_{kl} d a_{kl}}{(1 - b_{lk} d a_{kl}) d a_{kl}} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} b_{ik} b_{lj} y_j \right)}{\sum_{i=1}^{n} x_i} \tag{9}$$

Finally, reorganizing equation (9) yields the following expression:

$$\varepsilon_{x_\bullet a_{kl}}(d) = \frac{a_{kl} x_l \sum_{i=1}^{n} b_{ik}}{(1 - d a_{kl} b_{lk}) \sum_{i=1}^{n} x_i} \tag{10}$$

In this case, the most important coefficients are those whose elasticities reach the highest values.[3] Changes in these coefficients involve the greatest variations in the output of the economic system as a whole. One obvious means of obtaining such a measure is to calculate the inverse of equation (10), paralleling the TL approach:

$$r_{kl}^{x_\bullet} = \frac{1}{\varepsilon_{x_\bullet a_{kl}}(d)} \tag{11}$$

Assuming $d = 0.01$ (that is, 1% of the coefficient's variation), and that, for instance, $\varepsilon_{x_\bullet a_{kl}}(d) = 0.05$, and $r_{kl}^{x_\bullet} = 20$, this means that if a_{kl} varies by 20%, the overall output changes, ceteris paribus, by 1%.

From a policy point of view, a second extension follows. We have assumed that output is the best economic variable reference for measuring welfare. Nevertheless, in most instances, other variables are likely to hold more economic, social or environmental relevance, e.g. GDP, employment, CO_2 emissions, or imported energy supplies. Hence, it would be fundamental to formulate importance measures, using such benchmark variables. The alternative elasticity-based measure is proposed here in order to address this issue. Deriving the requisite expression is analogous to the case above. We need only identify e_i as the benchmark variable for activity branch i. We can then compute coefficients c_i that relate to the aforementioned variable and the sector's output as $c_i = e_i/x_i$. Considering now $e_i = c_i\Sigma_{l=1}^{n} b_{il}y_l$ expression (10) becomes:

$$\varepsilon_{e_\bullet a_{kl}}(d) = \frac{a_{kl}x_l \sum_{i=1}^{n} c_i b_{ik}}{(1 - da_{kl}b_{lk}) \sum_{i=1}^{n} c_i x_i} \tag{12}$$

The most important coefficients are those that have the highest value of elasticity. Changes in those coefficients will generate the greatest variations in the overall value of the benchmark variable.

This elasticity-based extension is an attempt to lend ICs more of a social welfare perspective. Nevertheless, complex scenarios in which coefficients and benchmark variables (output, employment) are subjected to a set of restrictions, or even a group of benchmark variables, might crop up. In such cases, the relevance of a coefficient is not only determined by the elasticity of the benchmark variable, but also by any limitations to free changes in coefficients. Such restrictions could be budgetary, political, or technological (for instance, the balance between certain inputs) in nature. Within this context, an approach that yields maximum flexibility could be quite valuable, since it would allow policy makers to evaluate scenarios that are closer to those that their world is likely to face. With the aforementioned challenge in mind, we turn to developing a framework based on linear programming.

4. An Alternative Approach based on Linear Programming

Dantzig (1949), Dorfman *et al.* (1958), among others, worked on mathematical programming within the input–output framework. Since these pioneering works, many applications have integrated the linear programming and input–output frameworks (recent applications include Bohlin and Widell, 2006; Sikdar *et al.*, 2006; Strømann and Duchin, 2006; ten Raa and Sahoo, 2007). The explicit ability to handle constraints within linear programming means that it lends itself to many possible applications that are parallel in nature to those of input–output analysis, not least of which is the set mentioned at the end of the previous section. This is largely because they can both handle the concept of production functions within an economic system.

Starting from equation (12), the quantity of the benchmark variable related to sector i ($=1, \ldots, n$) is given by the equality:

$$e_i = c_i\left(\sum_{j=1}^{n} a_{ij}x_j + y_i\right) \tag{13}$$

Taking all industries into account, the overall quantity of the benchmark variable will be:

$$e_{tot} = e_1 + e_2 + \cdots + e_n \tag{14}$$

If we assume an increase of the above quantity between two time periods 0 and 1, the quantity of the benchmark variable for the latter period will be:

$$e_{tot(1)} = e_{tot(0)} + \Delta e_{tot(0)} \tag{15}$$

If all of this increase is due to a change in the model's coefficients, the following applies for all i:

$$e_{i(1)} = c_{i(0)}\left(\sum_{j=1}^{n} a_{ij}^{*}x_{j(0)} + y_{i(0)}\right) \tag{16}$$

where the '*' refers to solution coefficients. The identities of the system of equations in equation (16) will not be fulfilled for all the n activity branches since there should be a change in at least one c_i coefficient. Supplementary variables (sv) will be added to pick up any errors in the system after the new benchmark levels are attained.[4] That is, for all i, j for which $a_{ij}(0) \neq 0$

$$a_{ij}^{*} \leq a_{ij(0)} + sv_{a_{ij}} \tag{17}$$

with

$$sv_{a_{ij}} \geq 0 \tag{18}$$

The minimization of the sum of all the sv will be proposed as the model's objective function:

$$\min \sum sv_{a_{ij}} \tag{19}$$

This objective function is subject to conditions (14)–(18).

When the supplementary variables $sv_{a_{ij}}$ are introduced, a feasible solution for the model is ensured, allowing us to analyze the sensitivity of the original coefficients. In order to do this, the model is solved one time (the *basic* solution). Then, the model is re-solved but with a coefficient a_{ij} selected along with its corresponding constraints. Thus, the coefficient becomes a variable. This allows the change of the coefficient to contribute to the model's objective function, which is to minimize the sum of the errors under the condition that there will be a global increase in the quantity of the benchmark variable. The linear program is modeled to apply itself to each coefficient in turn.

Let us now consider an iteration whereby constraints on a coefficient are relaxed in equation (17), so that its change contributes to objective function (19). If, after solving the model, a small change in the coefficient, with respect to its original value (at period 0), causes a large reduction in the value of objective function, then we consider it to be an IC.[5] This means that the coefficient demonstrates a high potential for enhancing the level of the benchmark variable, since a tiny variation in the coefficient could yield a great improvement.

Hence, the analysis could be interpreted as a search for coefficients that have the highest potential for improving conditions expressed in terms of the benchmark variable by means of the least effort – that is, by means of the smallest possible change within the current technological structure. The importance of the coefficients from the above perspective, can be quantified by the following indicator:

$$\delta^a_{ij} = \frac{(obj_{bas} - obj_{ij})/obj_{bas}}{(a_{ij(0)} - a^*_{ij})/a_{ij(0)}} \tag{20}$$

where: obj_{bas} is the value of objective function (19) in the case of the first (baseline) solution, were no coefficient variability is permitted, and obj_{ij} is the value of objective function (19). In this case, coefficient a_{ij} is unconstrained. This indicator is, therefore, simply an elasticity index. Thus, a high δ^a_{ij} means that the benchmark variable level is very sensitive to changes in the coefficient a_{ij}, which is then considered an IC.

Naturally, in the exposition above we have omitted constraints upon elements (coefficients, benchmark variable values...) or upon relationships among elements derived from the different economic development scenarios. All of these *ad hoc* restrictions, which are different from definitions within an input–output framework, could easily be added to the model. This is, of course, the main advantage of this approach. In the mathematical explanation above, we only draw up the basic specification of the model.

Regardless of the theoretical argument in favor of the approaches we introduce above, it may be that the TL approach provides similar results. This would heavily discount our articulation of the apparent theoretical deficiencies of TL. Thus, we must verify whether it yields similar or different results compared with alternatives that we know estimate the 'overall welfare' importance of a coefficients. With this in mind, we develop a simple application for locating the ICs using national employment in eight European countries. The results are described in the following section.

5. A Comparison among Alternative Approaches

In this section we compare rankings of coefficient importance using both TL and alternative approaches. We use the 1995 domestic input–output tables of eight European countries, which are taken from the Eurostat IO-Database (Eurostat, 2005). The tables embrace 30 economic industries/commodities, see the Appendix for a classification scheme. The aim of these tests is to verify whether the TL approach yields a viable measure of the concept of importance (understood as the contribution to 'overall welfare') despite the fact that it is clearly not properly equipped to do so. That is, we will compare results from the TL approach to those obtained via three alternative approaches, proposed in the previous two sections, to observe whether their classification of the importance of coefficients yields significantly different results. If they do yield very different results, this paper will have contributed to a body of literature that suggests using at least one of these alternatives instead of the TL approach, when the goal is to improve the overall welfare of an economy or to improve the accuracy of an economic model.

The employment level in each sector and country has been considered as the target variable. In the linear programming-based approach, an increase of 1% in the overall employment level has been included as an objective. Thus, coefficients have been ranked according to index (20). The first task in the application consists of measuring the level of global similarity inherent in the order of all the non-zero coefficients across approaches. Hence, Spearman's rank correlation coefficient is computed. It measures how closely two ranked lists reflect one another. That is, for all i, j for which $a_{ij}(0) \neq 0$ we measure:

$$r_s = 1 - \frac{6\sum_{i=1}^{n}\sum_{j=1}^{n} dif_{ij}^2}{m(m^2-1)} \qquad (21)$$

where m is the number of non-zero coefficients and *dif* is the difference between the positions reached by coefficient a_{ij} in either of the compared techniques. Spearman's rank correlation coefficient close to 1.0 means that the techniques being compared yield similar coefficient rankings. If the correlation coefficient is close to 0.0, it means that the lists rank few coefficients as being similar.

Table 1 shows the results. The second column provides Spearman's rank correlation between the TL approach and the first extension – the elasticity for the overall output (10), EO. Column 3 shows the rank correlation between the TL method and the second extension – the elasticity regarding the overall employment (12), EE. The fourth column shows the results of the rank correlation between results from TL and the linear programming approach, LP. The last two columns compare the suggested extensions in relation to one another.

The results show that the order of coefficients differs the most between the TL and LP approaches, followed by that between the TL and EE approaches. On the other hand, all of the approaches appear to yield fairly similar results, as far as ranking the coefficients in

Table 1. Comparison of 'Tolerable Limits' and alternative approaches. Spearman's coefficients for eight European countries

Country	TL – Elast. Output	TL–Elast. Employment.	TL–LP Approach	Elast. Output–Elast. Employment	Elast. Output–LP Approach.	Elast. Employment–LP Approach.
Austria	0.8840	0.8256	0.7972	0.9686	0.9486	0.9863
Denmark	0.8897	0.8501	0.8233	0.9854	0.9656	0.9941
Finland	0.8691	0.8437	0.8279	0.9833	0.9595	0.9900
Germany	0.8658	0.8411	0.8065	0.9878	0.9671	0.9936
Italy	0.9289	0.8954	0.8824	0.9832	0.9706	0.9961
Netherlands	0.8551	0.7992	0.7701	0.9627	0.9316	0.9938
Spain	0.9101	0.8675	0.8302	0.9786	0.9485	0.9903
Sweden	0.8302	0.7852	0.7598	0.9767	0.9543	0.9943

order of importance, as Spearman's correlation coefficients by country show. Then again, the alternative approaches yield ranked orders that are more similar to each other than TL is to any one of them.

Conclusions depend somewhat on the country being observed. Italy and Spain tend to have the highest values on Spearman's index, in any case where TL is compared with one of the other approaches. In both countries, it is likely that the locations of the ICs are rather homogeneous, regardless of the nature of the definition of importance. The lowest Spearman coefficient values between results from the TL approach and the other three methods are obtained for the Netherlands, Sweden and Austria.

The correlation coefficients across countries, as displayed in Table 1, yield few firm conclusions. Thus, we calculated a weighted index for each industry based on the coefficient rows.[6] This index was created by cutting the rank-ordered list of the 300 most important coefficients into 10 consecutive sections (that is, the first 30 most important coefficients appear in section 1, the second 30 most important coefficients in section 2, and so on). The percentage of coefficients belonging to a given industry in each section is weighted by the inverse order of that section (that is, if 15% of the coefficients in section 1 belongs to that industry, the score of such an industry with regard to this section is 0.15×1, whereas if 15% of the most important coefficients of section 2 belongs to the same industry, its score regarding section 2 is $0.15 \times 1/2$, and so on). Thus, the industries containing a high percentage of coefficients in the first sections are those that were of greater value in index terms. The maximum index value is set at 100 (a percentage). This value would be attained when all of an industry's 30 coefficients (remember that only 30 industries are in the EU tables) are included in the first section (which are among the input–output table's most important coefficients) and thus would have a weight of 1.0. In short, the index for industry i is:

$$II_i = \sum_{k=1}^{10} \frac{n_{i,k}}{30\,k} \times 100 \tag{22}$$

where: $n_{i,k}$ is the number of coefficients in industry i (by row) belonging to section k, which includes the coefficients for the interval $[(k-1)\times 30+1,\ k\times 30]$ in order of importance.

Table 2 shows, in the case of the sectors with the greatest importance index, the cases with the greatest differences between TL and all other approaches across all countries. It reveals two main differences among the various approaches with regard to the way that they assess the importance of industry coefficients. Namely, TL tends to give greater weight to the primary (agriculture and mining) and manufacturing industries, whereas the other approaches give greater weight to services. TL basic-industry bias compared with other approaches is especially apparent in the case of sectors: 3 Mining and quarrying of energy producing materials, 4 Mining and quarrying except energy producing materials, 10 Manufacture of coke, refined petroleum products and nuclear fuel, 12 Manufacture of rubber and plastic products, 13 Manufacture of other non-metallic mineral products, and 8 Manufacture of wood and wood products in all countries except Austria and Finland, where the EE approach offers a similar or higher sector assessment.

On the other hand, TL assesses the construction sector and services sectors with lower weighted scores: 21 Wholesale and retail trade…, 23 Transport, storage and communication and 25 Real estate, renting and business activities. Higher weighted scores by the EE and LP approaches are particularly notable for sector 21 Wholesale and retail trade… as compared to EO. This is undoubtedly due to the extreme labor-intense nature of that sector. The opposite is true of sector 25 Real estate, renting and business activities. In construction and transport, the assessment differences vary from country to country. Finally, we should note that sector 19 Electricity, gas and water supply is valued significantly more by TL and EO, which is probably due to the capital intensity of this activity.

Table 3 was drawn up to explain the underlying causes of the differences in the coefficient assessments in each approach. It shows the most important coefficients based upon the different approaches. Only the top ten coefficients for each approach are displayed. Table 3 classifies these coefficients into groups based upon relatively homogeneous assessment patterns across the various approaches. The 'Ranking' column shows the order obtained by each of the four techniques, TL, EO, EE and LP. A hyphen indicates that the specified coefficient is not among the ten most important. The next column shows the maximum variation reached by a given industry in its output or employment level, when the coefficient studied varies by 1%. In each case, as explained above, the industry is the selling sector involved in the coefficient, since the direct effects cause the variation in the sector to be significantly greater than in the rest of the other sectors. The next two columns indicate the relative weight or output and employment level proportion in the sector, compared with the total for the economy, respectively. The last two columns show the variation in output and the overall level of employment when the coefficient varies by 1%.

The results in Table 3 underline the causes of the differences in the order of coefficients produced by the various approaches. The first set of coefficients at the top of the table has far higher assessments from TL than by the rest. These coefficients, not surprisingly, are basically related to the sales of the mining and non-metal product industry to the energy and construction industries. All of the coefficients have the common characteristic that a 1% change to its value causes a change of over 0.3% in the output of the selling industry – a relatively high percentage. If the level of activity of the industries with regard to output

Table 2. Index of the sectors' importance based on the share of coefficients in intervals of importance. Main differences between approaches on the coefficients' assessment

	Selling sector	Austria				Denmark				Finland				Germany			
		TL	EO	EE	LP	TL	EO	EE	LP	TL	EO	EE	LP	TL	EO	EE	LP
3	Mining: ener.	13.7	1.3	1.2	1.1	6.7	2.8	1.2	0.7	8.4	1.1	1.1	0.8	13.6	3.7	3.4	3.4
4	Mining: non ener.	14.1	2.6	1.8	1.5	16.1	1.6	1.0	0.9	14.8	2.7	2.4	2.4	12.0	1.7	1.5	1.3
8	Manuf. of wood. . .	14.6	8.7	13.2	7.6	12.0	3.9	3.9	3.9	8.8	9.0	10.5	6.3	15.2	5.3	7.0	4.8
10	Coke, ref. petrol. . .	19.1	4.8	1.6	0.8	10.8	2.8	0.4	0.0	13.3	5.2	1.4	0.5	14.9	3.9	1.6	0.0
12	Manuf. rubber, plastic. . .	13.8	4.9	4.7	4.8	17.2	6.6	5.6	6.1	17.4	5.1	4.0	4.6	16.6	10.5	8.1	8.1
13	Non-metal. prods.	11.6	7.3	6.9	6.3	8.3	5.7	5.3	5.3	9.5	4.4	4.4	4.4	12.6	8.1	7.0	6.9
19	Electricity. . .	26.7	23.1	14.8	11.7	17.7	13.3	6.5	5.7	20.9	16.8	10.3	10.8	19.7	14.5	11.6	9.1
20	Construction	8.4	18.0	18.0	18.3	9.0	16.5	16.5	14.7	5.4	8.2	10.4	11.0	4.7	15.7	16.3	16.5
21	Trade	11.8	33.8	39.2	43.1	12.7	39.5	43.2	45.4	11.8	24.3	34.8	37.8	10.9	28.2	39.9	44.3
23	Transport. . .	19.4	27.3	29.5	33.2	17.5	31.1	29.7	28.1	17.2	30.7	34.0	39.3	16.2	21.7	25.7	25.8
25	Business act.	20.2	45.6	34.8	33.3	19.3	43.1	35.9	35.6	11.2	34.5	30.1	28.6	18.3	49.0	37.5	37.7
		Italy				Netherlands				Spain				Sweden			
		TL	EO	EE	LP	TL	EO	EE	LP	TL	EO	EE	LP	TL	EO	EE	LP
3	Mining: ener.	6.7	1.3	0.9	1.0	8.5	5.9	1.5	0.8	8.6	2.4	2.4	2.5	12.2	0.4	0.3	0.0
4	Mining: non ener.	15.0	1.4	1.5	1.6	13.3	1.4	1.1	0.3	12.9	2.5	2.4	2.4	11.7	2.3	1.6	1.6
8	Manuf. of wood. . .	15.9	6.5	8.2	7.6	17.7	2.8	3.2	3.3	18.3	7.1	7.5	7.5	16.9	9.4	9.1	5.7
10	Coke, ref. petrol. . .	14.1	4.3	1.0	0.9	10.8	4.9	1.3	0.7	13.4	4.2	0.8	0.0	11.1	2.2	0.3	0.0
12	Manuf. rubber, plastic. . .	18.7	10.5	8.6	8.4	11.9	4.2	3.4	3.4	21.6	8.9	7.8	7.1	14.1	5.2	3.9	3.9
13	Non-metal. prods.	12.4	8.3	8.5	8.3	9.5	5.3	5.1	5.1	9.4	7.2	6.9	6.7	12.7	3.1	2.8	2.6
19	Electricity. . .	20.1	12.7	7.6	7.6	19.9	20.6	7.9	6.6	21.3	18.4	10.4	8.0	18.1	13.6	7.7	7.1
20	Construction	7.9	12.7	15.3	15.5	10.8	16.9	17.7	16.1	6.0	15.0	16.3	16.3	8.8	12.3	14.2	16.3
21	Trade	9.1	23.4	35.7	42.1	10.1	27.8	37.2	39.9	8.7	23.5	35.7	41.7	10.3	24.4	38.0	44.9
23	Transport. . .	16.4	28.3	27.1	27.1	13.2	22.2	22.3	23.7	18.1	31.6	37.8	42.3	19.8	34.7	34.5	34.0
25	Business act.	17.1	36.5	32.8	33.9	18.6	45.0	46.6	46.8	14.1	36.5	31.8	30.1	17.0	51.6	43.8	36.1

Note: TL = tolerable limits approach, EO = elasticity regarding overall output, EE = elasticity regarding overall employment level, and LP = linear programming approach.

Table 3. Analysis of important coefficients

Co.		Seller		Buyer	Ranking (TL / EE / EO / LP)	% change seller's output / employment	Seller's weight in overall output	Seller's weight in overall employment	% change overall output	% change overall employment
		SET 1								
Dmk	2	Fishing	5	Manuf. of food...	3 - - -	0.4472	0.2487	0.2648	0.0017	0.0017
Aus	3	Mining: ener.	10	Coke, ref. petrol...	4 - - -	0.4121	0.0881	0.0512	0.0006	0.0004
Dmk	3	Mining: ener.	10	Coke, ref. petrol...	4 - - -	0.3320	0.5180	0.0378	0.0022	0.0005
Ita	3	Mining: ener.	10	Coke, ref. petrol...	1 - - -	0.5670	0.1735	0.0500	0.0011	0.0004
Aus	3	Mining: ener.	19	Electricity...	1 - - -	0.6186	0.0881	0.0512	0.0008	0.0006
Fin	3	Mining: ener.	19	Electricity...	1 - - -	0.7829	0.1344	0.0972	0.0018	0.0014
Ger	3	Mining: ener.	19	Electricity...	3 - - -	0.5318	0.4028	0.3932	0.0036	0.0033
Ita	3	Mining: ener.	19	Electricity...	3 - - -	0.4909	0.1735	0.0500	0.0009	0.0003
Net	3	Mining: ener.	19	Electricity...	3 - - -	0.4440	1.5663	0.0979	0.0079	0.0013
Spa	3	Mining: ener.	19	Electricity...	1 - - -	0.7516	0.2388	0.2137	0.0026	0.0021
Swe	3	Mining: ener.	19	Electricity...	3 - - -	0.3740	0.0340	0.0244	0.0002	0.0002
Spa	4	Mining: non ener.	13	Non-metal. prods.	5 - - -	0.4495	0.2622	0.1547	0.0020	0.0013
Fin	4	Mining: non ener.	20	Construction	5 - - -	0.3776	0.2913	0.1944	0.0018	0.0013
Ger	4	Mining: non ener.	20	Construction	5 - - -	0.3754	0.2036	0.1284	0.0013	0.0009
Spa	7	Leather prods.	7	Leather prods.	4 - - -	0.6185	0.7984	0.6042	0.0073	0.0058
Ita	8	Manuf. of wood...	18	Manuf. n.e.c.	5 - - -	0.4259	0.8125	0.9229	0.0047	0.0049
Ger	8	Manuf. of wood...	20	Construction	4 - - -	0.4046	0.7397	0.6046	0.0044	0.0038
Net	8	Manuf. of wood...	20	Construction	4 - - -	0.3763	0.3545	0.2798	0.0019	0.0016
Dmk	9	Pulp, paper...	25	Business act.	5 - - -	0.2861	2.2754	2.0045	0.0086	0.0075
Dmk	13	Non-metal. prods.	20	Construction	2 - - -	0.5824	1.0086	0.7564	0.0087	0.0068
Fin	13	Non-metal. prods.	20	Construction	2 - - -	0.5553	0.7961	0.6803	0.0068	0.0056
Net	13	Non-metal. prods.	20	Construction	1 - - -	0.5531	0.8437	0.5316	0.0068	0.0047
Swe	13	Non-metal. prods.	20	Construction	1 - - -	0.3963	0.6127	0.4385	0.0039	0.0029
		SET 2								
Spa	5	Manuf. of food...	5	Manuf. of food...	- 4 4 8	0.2336	7.1306	3.0433	0.0293	0.0239
Spa	20	Construction	20	Construction	- 3 3 2	0.2067	9.9114	9.1150	0.0331	0.0288
Aus	20	Construction	25	Business act.	- 5 5 3	0.1156	8.5976	7.3258	0.0142	0.0118
Fin	20	Construction	25	Business act.	- 5 7 7	0.1555	5.4831	5.7337	0.0151	0.0145

Ger	20	Construction	25	Business act.	- 7 5 5	0.0947	7.8962	8.6325	0.0128	0.0124
Spa	20	Construction	25	Business act.	- 6 6 5	0.0794	10.8038	5.9391	0.0110	0.0068
Dmk	23	Transport. . .	21	Trade	- 5 4 4	0.1597	8.9298	6.8079	0.0175	0.0137
Dmk	23	Transport. . .	23	Transport. . .	- 4 3 2	0.1676	8.9298	6.8079	0.0183	0.0143
Ger	23	Transport. . .	23	Transport. . .	10 3 3 3	0.2609	5.5818	5.7755	0.0185	0.0183
Swe	23	Transport. . .	23	Transport. . .	- 2 2 4	0.1844	8.0412	6.8210	0.0199	0.0165
Aus	25	Business act.	21	Trade	- 1 6 4	0.1203	12.8516	6.5061	0.0193	0.0112
Dmk	25	Business act.	21	Trade	- 3 5 5	0.1151	13.7683	7.9425	0.0200	0.0130
Ger	25	Business act.	21	Trade	- 2 7 7	0.0949	17.6381	8.4212	0.0193	0.0107
Ita	25	Business act.	21	Trade	- 2 4 4	0.1338	12.5361	7.6878	0.0203	0.0134
Net	25	Business act.	21	Trade	- 5 4 4	0.0905	13.2578	13.0946	0.0148	0.0144
Aus	25	Business act.	25	Business act.	- 2 7 5	0.1190	12.8516	6.5061	0.0191	0.0111
Ger	25	Business act.	25	Business act.	- 1 2 2	0.1718	17.6381	8.4212	0.0350	0.0194
Ita	25	Business act.	25	Business act.	- 4 6 5	0.1258	12.5361	7.6878	0.0191	0.0126
Net	25	Business act.	25	Business act.	- 3 3 2	0.1293	13.2578	13.0946	0.0212	0.0206
Swe	25	Business act.	25	Business act.	- 1 1 3	0.1501	18.8869	8.7698	0.0373	0.0220
		SET 3								
Aus	1	Agriculture. . .	5	Manuf. of food. . .	2 4 1 1	0.6029	2.0152	14.7797	0.0160	0.0920
Dmk	1	Agriculture. . .	5	Manuf. of food. . .	1 1 1 1	0.6907	3.7179	4.3495	0.0364	0.0390
Fin	1	Agriculture. . .	5	Manuf. Of food. . .	4 4 1 1	0.4018	3.9274	7.8231	0.0218	0.0364
Ger	1	Agriculture. . .	5	Manuf. of food. . .	2 5 1 1	0.5755	1.4014	2.9640	0.0130	0.0212
Ita	1	Agriculture. . .	5	Manuf. of food. . .	2 5 1 1	0.5521	2.2477	5.8602	0.0157	0.0346
Net	1	Agriculture. . .	5	Manuf. of food. . .	2 2 2 1	0.5026	3.5943	4.0011	0.0268	0.0258
Spa	1	Agriculture. . .	5	Manuf. of food. . .	3 1 1 1	0.6470	3.6632	7.3908	0.0358	0.0560
Swe	1	Agriculture. . .	5	Manuf. of food. . .	2 8 4 3	0.3895	2.1558	3.1181	0.0114	0.0145
Ita	6	Textil prods.	6	Textile prods.	4 1 2 2	0.4293	3.7683	3.7234	0.0213	0.0200
Fin	9	Pulp, paper. . .	9	Pulp, paper. . .	6 1 2 4	0.3460	9.4573	3.5471	0.0484	0.0269
Spa	13	Non-metal. prods.	20	Construction	2 5 5 4	0.6634	1.7656	1.2453	0.0202	0.0144
Swe	14	Metal prod.	14	Metal prod.	4 3 5 8	0.2734	4.5459	2.5335	0.0166	0.0106
Swe	20	Construction	25	Business act.	6 5 3 1	0.2312	4.3396	5.2132	0.0155	0.0163
		SET 4								
Aus	19	Electricity. . .	19	Electricity. . .	3 3 - -	0.4735	3.2930	0.9990	0.0180	0.0067
Net	19	Electricity. . .	19	Electricity. . .	5 7 - -	0.3022	2.7586	0.6016	0.0117	0.0031

(Table continued)

Table 3 *Continued*

Co.		Seller		Buyer	Ranking (TL / EE / EO / LP)	% change seller's output / employment	Seller's weight in overall output	Seller's weight in overall employment	% change overall output	% change overall employment
		SET 5								
Aus	1	Agriculture. . .	8	Manuf. of wood. . .	- - 4 2	0.0865	2.0152	14.7797	0.0023	0.0132
Fin	1	Agriculture. . .	9	Pulp, paper. . .	- - 8 5	0.1380	3.9274	7.8231	0.0075	0.0125
Ger	21	Trade	21	Trade	- - 6 4	0.0625	9.9892	15.3603	0.0088	0.0114
Net	21	Trade	21	Trade	- - 6 5	0.0484	10.9035	16.5921	0.0070	0.009

Note: column 4 contains the ranking of the corresponding coefficient regarding each approach according to the following order: TL approach, EO elasticity regarding overall output, EE elasticity regarding overall employment level, and LP approach. The hyphen means that the coefficient does not belong to the top-ten ranking considering the corresponding approach.

or employment is considered, however, their participation is generally low – no more than 1% for either indicator. Clearly, these coefficients affect the level of output and employment in other industries through their effects as a multiplier. In practice, however, these effects are small because the selling industry itself is small. As a result, variations in output and employment (the last two columns in Table 3) barely reach 0.01%.

In short, TL identifies the greatest dependencies between selling sectors and their corresponding buying sectors. Thus, production in sector 3 Mining and quarrying of energy producing materials depends heavily upon purchases made by energy-producing industries 19 Electricity, gas and water supply and 10 Manufacture of coke, refined petroleum products and nuclear fuel, 12 Manufacture of rubber and plastic products; likewise, industries 4 Mining and quarrying except energy producing materials and 13 Manufacture of other non-metallic mineral products, which includes the manufacture of cements and raw materials for construction, depend heavily on sector 20 Construction.

The opposite is true in the second group of coefficients. These are coefficients that TL does not classify among the most important, but which the other approaches do. This is clear in the column ‘maximum variation reached by the selling sector’, which shows that a 1% change to the coefficient alters the output of the selling sector by a mere 0.2%. Thus, TL does not place as much importance on these coefficients as it does on the previous groups. On the other hand, this selling industry has significant weight in the overall output and employment level, at over 5.5%, exceeding 17% of the total output in the case of sector 25 Real estate, renting and business activities in Germany and Sweden. This means that the variation of these coefficients does tend to change the overall output and employment level in a significant way, and well in excess of 0.01% in most cases. This is reflected in the high level of importance assigned to them by the elasticity and linear programming approaches. These coefficients basically represent sales in construction sales sector 25 Real estate, renting and business activities, and, in turn, from the latter to itself and to sector 21 Wholesale and retail trade. . ., as well as transport sales to itself. In other words, these are transactions between service sectors that are characterized by high participation in the overall output of the countries considered and that depend heavily upon labor as a factor of production.

The third group of coefficients includes several examples of coefficients valued as important both by TL and by the rest of the approaches. The main case in all countries is the coefficient that represents agricultural sales to the food industry. This clearly is a highly co-dependent relationship, since most agricultural products are almost exclusively destined to be processed as food. Thus, TL values this dependency by assigning to this group of coefficients the tendency to comprise a relatively high share of total output, varying between 2% and 4% by country. They are also industries that are relatively labor-intensive, so that the employment contribution is also comparatively high. Therefore, a change in a coefficient entails significant changes to output and the employment level in the country, in excess of 0.01% in both cases, so the alternative assessment methods also assign a high level of importance to the sector.

Finally, sets 4 and 5 show coefficients with opposing assessment patterns. While set 3 includes coefficients with higher importance assigned to them by TL and EO, set 4 had higher assessments in EE and LP. These examples illustrate the change to the results that can come from the need to take into account a reference variable that differs from the output. Thus, the coefficients in set 4 represent transactions in which the selling sector has a low coefficient in the intensity of use of labor factor, so, although the contribution to the overall output may be relatively high, it is not as much as regards the level of employment. The opposite is true for the coefficients in set 5. The selling sector (agriculture and trade) is very labor-intensive, so these coefficients are valued more highly by the approaches that account for the level of employment.

Summarizing the results, in most countries the same general pattern in valuing the importance of the coefficients in an input–output table is obtained when using the TL method and the other approaches in which the concept of 'importance' more squarely pertains to the concept of general welfare. The difference between TL and the other approaches is focused on TL's 'preference' towards coefficients that deal with sales of primary industries and manufacturing, and its lower assessment of the services industries. These differences are especially accentuated in sectors with extremes in labor (lower for TL) and capital intensity (higher for TL).

The main difference between TL and the other approaches is that TL does not account for a coefficient's contribution to the overall economic system; rather, its concept of importance is related to the intensity of the dependence of a selling sector on the industries it buys from directly. In essence, it does not measure importance in reference to other industries in the economy.

6. Concluding Remarks

The location of important coefficients (ICs) in an input–output framework was one of the first applications of Sherman and Morrison (1950, S&M hereafter). Among the many branches of research that have stemmed from the S&M article, the tolerable limits (TL) approach is perhaps the one most cited. Curiously, the concept of 'importance' given to coefficients through TL seems unsuitable for most practical purposes. TL defines an important coefficient as one that, when changed, brings about a significant change (usually 1%) in at least one sector. Due to the configuration of Leontief's inverse, it is generally the selling sector that is the sector associated with the identified coefficient.

This incongruity of the TL approach raises two questions from a policy point of view. The first relates to the political economic concept of social welfare. That is, a coefficient might be TL-important but have little impact economy-wide. The second relates to the economic variable used to classify coefficients. The literature on TL explicitly and repeatedly makes exclusive reference to output by industry. Nevertheless, from a policy perspective, it would seem more valuable to use almost any other economic variable, and preferably one with an explicit social weight, like GDP, jobs, energy usage, or pollution generation.

Accordingly, we suggest a re-formulation after reconsidering key points from S&M. It applies the concept of elasticity of certain pre-specified benchmark variables to the variation of a direct coefficient. We also suggest a second alternative approach based on linear programming (LP). The economic interpretation of its results may be no clearer than those obtained through TL or the elasticity approach. Instead, it has the appeal of a more flexible framework for dealing with the broad set of restrictions that tend to arise in this setting: those pertaining to technological change, budget restrictions, and so on. Thus, the LP approach should be better able to identify ICs when coefficients have a limited ability to change.

Finally, while from a theoretical perspective the TL approach may hold limited appeal, it may well be that from a practical perspective it yields results that are the same or at least similar to those obtained by approaches that actually measure importance from a 'social welfare' perspective. If so, any objections to the TL approach may be mitigated somewhat, especially if the approach is easier to implement.

Our findings suggest that the TL does in fact yield results that are relatively similar to other approaches considered in this paper, at least with regard to the ranking of direct coefficients in order of importance. Of course, this could be a function of the level of aggregation of the input–output tables and of the structure/nature of the economies employed in the analysis. Nonetheless, of the approaches evaluated here, the TL approach yielded an ordering of direct coefficient importance that, while similar, was also somewhat distinct from that provided by the other approaches. This is largely because it does not take sector size or factor intensity into account.

In conclusion, it is appropriate to use TL when one is searching for coefficients that have the greatest potential for changing the output of a sector. In this regard, it appears to have a bias toward selecting primary and manufacturing industries. But it is not the best choice if one's aim is to identify direct coefficients that, if changed, would improve social welfare. In this case, the elasticities-based or linear-programming approaches are clearly more suitable from both theoretical and empirical perspectives. Moreover, the LP approach should, in theory, be preferred when more complex scenarios are employed.

To sum up, this paper investigates several alternative ways to identify ICs, all of which are more or less similar. A prime difference among them, however, is the definition of 'importance' that underlies them. The concept 'important coefficient' alone is, from our point of view, generally inadequately articulated. There are many alternative ways to understand 'importance' with regard to a direct coefficient in an input–output analysis. Therefore, it is critical that researchers make their understanding of the concept of 'importance' perfectly clear, especially when they use formulae that claim a strong link to the S&M formula.

Notes

[1]See Dietzenbacher and Lahr (2001) for an overview of the literature.

[2]See Percoco *et al.* (2006) for a recent example of this type of application.

[3]In order to distinguish important coefficients from unimportant ones, we need a threshold, which will be necessarily arbitrary. An alternative option is to consider the coefficients in terms of degrees of relative importance alone.

[4]'Errors' are understood here as mismatches in the IOT identities.

[5]On the contrary, the coefficients can be considered as unimportant if a large modification in the coefficient is proposed, but only a small reduction in the objective is obtained.

[6]The index by columns was also calculated, although the results obtained in this case were less conclusive. This is probably due to the fact that the importance definition in the TL formula is related to the influence of a coefficient on the seller sector (row), as has been indicated.

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Appendix. R-31 Activity Branches and Aggregates of the Final Demand

1	Agriculture, hunting and forestry
2	Fishing
3	Mining and quarrying of energy producing materials
4	Mining and quarrying except energy producing materials
5	Manufacture of food products; beverages and tobacco
6	Manufacture of textiles and textile products
7	Manufacture of leather and leather products
8	Manufacture of wood and wood products
9	Manufacture of pulp, paper and paper products; publishing and printing
10	Manufacture of coke, refined petroleum products and nuclear fuel
11	Manufacture of chemicals, chemical products and man-made fibers
12	Manufacture of rubber and plastic products
13	Manufacture of other non-metallic mineral products
14	Manufacture of basic metals and fabricated metal products
15	Manufacture of machinery and equipment n.e.c.
16	Manufacture of electrical and optical equipment
17	Manufacture of transport equipment
18	Manufacturing n.e.c.
19	Electricity, gas and water supply
20	Construction
21	Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods
22	Hotels and restaurants
23	Transport, storage and communication
24	Financial intermediation
25	Real estate, renting and business activities
26	Public administration and defense; compulsory social security
27	Education
28	Health and social work
29	Other community, social, personal service activities
30	Activities of households
FISIM	Financial brokerage indirectly measured
FCEH	Final consumption expenditures by households
FCEN	Final consumption expenditures by non-profit organizations serving households
FCEG	Final consumption expenditures by government
GFCF	Gross fixed capital formation
CIV	Changes in inventories and valuables
EXP	Exports